The Use of Upward Price Pressure Indices in Merger Analysis

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Before Professors Joseph Farrell and Carl Shapiro became chief economists at the FTC and DOJ, respectively, they wrote a paper proposing a new measure for evaluating potential unilateral effects in merger cases involving differentiated products.¹ This new measure—called Upward Pricing Pressure, or UPP—has been discussed in the Merger Guidelines review process that is now underway at the two Agencies.² If the Agencies decide to revise the Guidelines, it is possible that the new Guidelines might prescribe the use of the UPP (or some other price pressure index) for gauging the unilateral effects of horizontal mergers.³

Some commentators have suggested that the UPP methodology has limited practical use because it assumes a particular type of industry structure and because it is not closely related to the traditional market definition-market power analysis. This article explains why the UPP approach, and a variant termed GUPPI, are both useful and defensible.⁴

One can conceptualize the potential unilateral effects of a merger on consumer prices as a conflict between two opposing forces of upward and downward pricing pressure. The elimination of competition between the merging firms generates upward pricing pressure.⁵ The efficiency benefits from the merger generate downward pricing pressure.⁶ This approach to unilateral effects

⁴ Farrell and Shapiro recently released a revised version of their UPP paper, in which they briefly describe the GUPPI test proposed by Salop and Moresi as being “one sensible way to proceed.” Farrell & Shapiro, Economic Alternative to Market Definition, supra note 1, at 20. For the GUPPI test proposed by Salop and Moresi, see Steven C. Salop & Serge Moresi, Updating the Merger Guidelines: Comments (Public Comment to Horizontal Merger Guidelines Review Project Nov. 2009), available at http://www.ftc.gov/os/comments/horizontalmergerguides/545095-00032.pdf.
⁵ The upward pricing pressure depends on the closeness of substitution between the products of the merging firms relative to other products, as reflected by the magnitude of the diversion ratio between the merging firms’ products. (The diversion ratio from Product 1 to Product 2 is the sales volume gained by Product 2 following a unilateral increase in the price of Product 1, divided by the sales volume lost by Product 1 as a result of that price increase.) The upward pricing pressure also depends on the intensity of competition that the merging firms face in the marketplace (including competition between them), as reflected by the firms’ price-cost margins.
⁶ For cost-savings, the downward pricing pressure involves the nominal price. For quality improvements, it involves the quality-adjusted price.
analysis—based on upward and downward pricing pressure—is particularly well-suited to situations where firms engage in Bertrand competition, i.e., they sell differentiated products and compete mainly by setting the prices of their products.\(^7\)

In this article, I extend the Farrell-Shapiro UPP methodology in two ways. First, I explain how the UPP test can be applied usefully in situations where firms engage in either Cournot competition or bidding competition, as well as Bertrand competition (which served as the analytic underpinning of the Farrell-Shapiro UPP test). I then explain how the GUPPI (a modified version of the UPP measure) can be related directly to recent market definition tests, and thus could be used to create presumptions in the same way that the current Guidelines describe presumptions using market share statistics. The Appendix describes the derivations of all the formulas presented in this paper.

I. Applying the UPP Methodology to Different Industry Structures

A. Bertrand Competition. The UPP methodology proposed by Farrell and Shapiro is based on the Bertrand model of price competition among suppliers of differentiated products.\(^8\) This methodology, therefore, is most useful when it is applied to merger cases where firms compete mainly by posting prices and then supply the quantities that customers demand at those prices. The Bertrand model assumes that firms have sufficient excess capacity (or face relatively low barriers to capacity expansion), so that prices drive quantities.

The UPP test evaluates the net effect of two forces that work in opposite directions—namely, the upward pricing pressure from the elimination of competition between the merging firms, and the downward pricing pressure from the efficiencies generated by the merger. Under Bertrand competition, a merger of Firm 1 and Firm 2 creates (net) upward pricing pressure on Product 1 (i.e., the product sold by Firm 1) if the following inequality is satisfied:

\[
DR_{12} \times M_2 > E_1
\]

where \(DR_{12}\) denotes the diversion ratio from Product 1 to Product 2, \(M_2\) denotes the price-cost margin of Product 2 (in \$/unit), and \(E_1\) denotes the merger-induced variable cost savings for Product 1 (in \$/unit).

For example, suppose that the variable cost of producing Product 2 is $6 per unit and Firm 2 charges a price of $10 per unit for Product 2. Thus, the price-cost margin of Product 2 is $4 per unit, i.e., \(M_2 = 4\). Suppose that the variable cost of producing Product 1 is $7 per unit, and assume 10% variable cost savings post-merger. The cost savings for Product 1 thus will equal $0.70 per unit, i.e., \(E_1 = 0.7\). Finally, suppose that the diversion ratio from Product 1 to Product 2 is 20%, i.e., \(DR_{12} = 0.2\). In this example, the merger creates upward pricing pressure on Product 1 (since \(0.2 \times 4 = 0.8 > 0.7\)).

Note that the UPP test must be performed twice, once for each product. The UPP formula evaluates whether the merged firm would have an incentive to raise the price of Product 1 all else

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\(^7\) There can be significant unilateral concerns for products that are not each other’s closest substitutes. For example, suppose that Gerber (with approximately 70 percent of baby food sales) was a closer substitute for both Beech-Nut and Heinz than the two smaller brands were to each other. That fact obviously would not obviate unilateral effects concerns in a Heinz/Beech-Nut merger. Similarly, if Office Max was a closer substitute for both Staples and Office Depot than the two brands were for each other, that would not have eliminated unilateral concerns from a Staples/Office Depot merger.

equal. The same formula (but with the roles of Product 1 and Product 2 reversed) can be used also to evaluate the merger effect on pricing incentives for Product 2.\footnote{\ Ref.\n}

A higher diversion ratio or a higher margin (or both) tend to create greater upward pricing pressure post-merger for the following reason. When one of the two merging firms raises price unilaterally, a fraction of the sales that the firm loses are actually not lost because they are diverted to and thus recaptured by the merging partner. The higher the diversion ratio or the higher the margin of the merging partner, the greater the profits that are recaptured by the merging partner and thus the greater the incentive to raise price.

Merger-induced efficiencies in the form of variable cost savings tend to create downward pricing pressure, and thus the above formula essentially compares the incentive to raise price due to the “diversion effect” and the incentive to reduce price due to cost savings. The UPP test thus evaluates the net effect on pricing incentives.

**B. Cournot Competition.** Under Cournot competition, quantities drive prices—that is, each supplier first decides how much output to produce (or how much production capacity to install) and then prices adjust to ensure that demand equals supply.\footnote{\ Ref.\n} This is different from Bertrand competition where prices drive quantities. However, the UPP methodology also can be applied usefully under Cournot competition.\footnote{\ Ref.\n} This is true for firms selling homogeneous products, such as newsprint and iron ore, as well as firms selling differentiated products, such as large passenger aircraft and collectible wine.

In cases where firms engage in Cournot competition, the UPP test can be implemented using the above formula \textit{provided} that one uses a definition of “diversion ratio” that is different from that used in the case of Bertrand competition. Specifically, the same UPP formula still applies if one uses the “price diversion ratio” instead of the “quantity diversion ratio.”

To illustrate the difference between the price diversion ratio and the quantity diversion ratio, consider a hypothetical example using British Airways and Virgin Atlantic, and assume that these two airlines have exclusive rights to fly the London-Washington route.

When the demand for international travel is low, British Airways and Virgin Atlantic are likely to have many empty seats (excess capacity) and charge relatively low airfares. In this environment, one might assume that the two airlines behave according to the Bertrand model. Suppose that British Airways unilaterally were to increase its airfares and lose 100 passengers on its London–Washington flights. Suppose also that 60 out of those 100 passengers would switch to Virgin Atlantic (while the remaining 40 passengers might decide to switch to connecting flights, or to fly to other cities using different airlines, or not travel at all). In this example, the quantity diversion ratio is 60 percent (i.e., 60 divided by 100).\footnote{\ Ref.\n} This is the relevant diversion ratio to use in the Bertrand model, because the Bertrand model assumes that Virgin Atlantic would use its excess capacity (which might involve increasing the number of flights on that route) to accommodate the additional demand from those 60 passengers. Note that, if the two airlines were perfect substitutes
for one another (i.e., if they offered homogeneous services), then all the passengers lost by British Airways would switch to Virgin Atlantic, in which case the diversion ratio would be 100 percent.

When the demand for international travel is strong, and the number of passengers that British Airlines and Virgin Atlantic can carry is constrained by the airlines’ production capacity (including the number of landing slots available at the two airports), airfares are likely to be relatively high. In this environment, one might assume that the two airlines behave according to the Cournot model. Suppose that British Airways were to eliminate one of its London–Washington flights (and use the two landing slots for other routes). The passengers of that flight would ask British Airways and Virgin Atlantic for seats on the other flights on that route. Since these other flights are most likely to be sold out, the increased demand for those flights would lead British Airways and Virgin Atlantic to raise airfares on those flights (while keeping them sold out). If passengers view British Airways and Virgin Atlantic as differentiated airlines, then the passengers of the eliminated flight would have a preference for staying with British Airways. This implies that Virgin Atlantic would raise airfares by a smaller amount than British Airways. 13

Suppose that Virgin Atlantic would raise airfares by $60 per ticket. If instead the eliminated flight had been a Virgin Atlantic flight (not a British Airways flight), then Virgin Atlantic would have been able to raise airfares (on its remaining flights on that route) by more than $60—say, by $100 per ticket. In this example, the price diversion ratio is 60 percent (i.e., $60 divided by $100). 14

Note that, if the two airlines were perfect substitutes for one another (i.e., if they offered homogeneous services), then Virgin Atlantic would raise airfares by the same amount regardless of whether the eliminated flight would be a British Airways flight or a Virgin Atlantic flight. If so, the diversion ratio would be 100 percent.

This hypothetical example suggests that the UPP test can in principle be applied under Cournot competition as well as under Bertrand competition. The formula described in section I.A above can be used directly, provided that the diversion ratio is defined in terms of prices rather than quantities.

In either case, the relevant diversion ratio in principle can be estimated using econometric techniques or other evidence. In the early stages of a merger investigation, the relevant diversion ratio might be estimated using customer surveys or natural experiments. 15

C. Bidding Competition. As explained above, the basic UPP methodology is based on the standard Bertrand model where suppliers post prices and customers choose which supplier(s) to buy from based on the posted prices. The UPP methodology also can be applied usefully in situations where suppliers set prices through bidding competition. 16 For example, in wholesale and intermediate goods industries, large buyers often select their suppliers using procurement auctions, and suppliers compete for each customer account separately. In these situations, an auction

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13 If the two airlines were completely differentiated, no British Airways passenger would be willing to switch to Virgin Atlantic, and thus the elimination of a British Airways flight would not lead to any increase in Virgin Atlantic’s airfares. In this case, the price diversion ratio would be zero.

14 In this example, it happens that the price diversion ratio (when demand for traveling is high) is equal to the quantity diversion ratio (when demand for traveling is low). In general, the two different diversion measures might not be equal, and either one could be larger.

15 It might also be possible to estimate the relevant diversion ratio by using shares within some group of products and by adjusting the estimate downward based on some assumed value of the loss to products outside the group.

16 For mathematical details, see Serge Moresi, Bidding Competition and the UPP Test (Public Comment to Horizontal Merger Guidelines Review Project Nov. 2009), available at http://www.ftc.gov/os/comments/horizontalmergerguides/545095-00040.pdf. For a non-technical introduction to bidding competition models, see, for example, Paul KLEMPERER, AUCTIONS: THEORY AND PRACTICE (2004).
model might be used instead of a Bertrand or Cournot model.

Customers generally seek competitive bids from several suppliers.\(^\text{17}\) In some cases, customers use a single round of bidding—i.e., they ask suppliers to submit “sealed bids” containing their contract offers—and select a supplier based on the best contract offered. This type of bidding mechanism is referred to as the (first-price) sealed-bid auction. In other cases, customers use two or more rounds of bidding to “negotiate” price, and inform bidders about the current best offer to reduce the prices that are bid in the next round. This type of bidding mechanism is referred to as the open-bid auction.

When bidding competition resembles a sealed-bid auction, the UPP test can be implemented using the above formula, provided that one uses a definition of “diversion ratio” that is different from that used in the context of standard Bertrand competition. Specifically, the above formula still applies if one uses the “winning probability diversion ratio” instead of the “quantity diversion ratio.”

For example, consider a hypothetical sealed-bid competition for a large customer. When a supplier contemplates whether it should increase its (sealed) bid, it assumes that all the rival bids submitted by the other suppliers would not change. Therefore, the bidder takes into account that its higher bid would reduce the probability that it would win the customer’s account, and thus increase the winning probability of each of the other firms. Following a unilateral increase in Firm 1’s bid, the winning probability diversion ratio from Firm 1 to Firm 2 is equal to the increase in Firm 2’s winning probability divided by the decrease in Firm 1’s winning probability. Intuitively, it is the fraction of the reduction in Firm 1’s winning probability that is captured by Firm 2. (For example, if Firm 2’s winning probability increases from 20 percent to 24 percent, and Firm 1’s winning probability decreases from 30% to 14%, then the diversion ratio equals 25 percent (i.e., 4% divided by 16%).) Having defined the diversion ratio in these terms, the UPP test in section I.A above can be implemented directly.

While the quantity and winning probability diversion ratios differ, the estimation of their magnitude might not require significantly different analyses. In many cases, the winning probability diversion ratio is estimated across a number of bidding competitions by comparing total gains and losses in terms of customers or accounts, not in terms of the bidders’ probabilities of winning a given single account.

When bidding competition resembles an open-bid auction, the UPP test can be approximated using the above formula and a “diversion ratio” based on market shares. Specifically, the diversion ratio to be used in the UPP formula is the market share of the merging partner divided by the total market share of the non-merging firms. For example, if the market shares of Product 1 and Product 2 are 25 percent and 15 percent, respectively, then the above UPP formula should be implemented using a diversion ratio from Product 1 to Product 2 equal to 25 percent (i.e., 15% divided by (100%-25%-15%)).\(^\text{18}\)

\(^{17}\) Each procurement may involve competition that is entirely distinct from that in other procurements, and thus each procurement may be considered a separate and distinct relevant market under certain conditions. These conditions include situations where products are customized according to the specifications of the particular buyer, and where such customization makes arbitrage infeasible. \textit{See, e.g.}, Competitive Impact Statement § II(B)2, United States v. Ingersoll-Dresser Pump Co., No. 1:00CV001818 (D.D.C. filed July 28, 2000).
D. Caveats. Failing the UPP test is relevant “circumstantial” evidence of adverse unilateral effects. It is not a proof that the proposed merger likely would lead to a substantial lessening of competition. It is only one factor among others. In particular, the UPP test does not account for several other factors that are potentially important for evaluating the likely competitive effects of a merger, including potential supply-side responses (i.e., entry and repositioning), the multi-product nature of many firms (i.e., the impact on pricing incentives of the merging firms’ sales of other substitutable or complementary products), potential pricing interdependencies (i.e., how the merging firms’ pricing initiatives might trigger particular responses by other firms), and dynamic factors such as network effects and learning by doing. The UPP test, therefore, is a useful screen and might be used as supporting evidence, but it is not a complete analysis of all the relevant factors.

II. Relating the UPP Methodology to Market Definition
A. The Gross Upward Price Pressure Index (GUPPI). The UPP test measures net upward pricing pressure (i.e., the upward pricing pressure from the merger versus the downward pricing pressure from the merger-induced efficiencies). In contrast, the GUPPI measures only the upward pricing component before netting out the downward pricing pressure from efficiencies. To make the connection to market definition clear, the GUPPI imposes a bit more structure on the analysis and expresses this upward pricing pressure as a percentage of the pre-merger price.

Formally, the GUPPI for Product 1 is calculated using the following formula:

$$GUPPI_1 = DR_{12} \times m_2 \times \frac{P_2}{P_1}$$

where 

- $DR_{12}$ denotes the diversion ratio from Product 1 to Product 2,
- $m_2$ denotes the variable profit margin of Product 2 as a fraction of revenue, and
- $P_2 / P_1$ denotes the relative price of Product 2 (relative to Product 1).

For example, if $DR_{12} = 20\%$, $m_2 = 50\%$, and $P_1 = P_2$, then $GUPPI_1 = 10\%$. The GUPPI for Product 2 is defined similarly as $GUPPI_2 = DR_{21} \times m_1 \times \frac{P_1}{P_2}$. Thus, $GUPPI$ is higher when either the diversion ratio to the merging partner’s product is higher, or the profit margin of the merging partner’s product is higher, or the relative price of the merging partner’s product is higher (or all three).

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18 In the open-bid auction model, the winner is the firm with the lowest cost of supplying the customer, and the equilibrium price paid by the customer is equal to the second-lowest cost. When a firm becomes more efficient and can supply customers at lower costs, the existing customers of that firm do not benefit from the cost savings, because the price they pay is determined by the cost of a rival firm. Similarly, the customers of the merging partner will not benefit either, because the firm will no longer compete against its merging partner. Thus, efficiencies create downward pricing pressure only for the customers of the non-merging firms. This roughly explains why the diversion ratio should be approximated by the market share of the merging partner divided by the total market share of the non-merging firms. See Moresi, supra note 16.

19 The UPP test takes into account merger-specific efficiencies by crediting the merging firms with a reduction (of, say, 10 percent) in their variable costs of production. Efficiencies also can take the form of product quality increases and faster innovation. It is unclear whether these other types of efficiencies can be accounted for in the formal analysis with “equivalent” percentage cost reductions. An alternative might be to credit the merging firms with cost reductions equal to, say, 10 percent of the pre-merger price.

20 The upward pricing pressure component in the UPP test (i.e., the term $DR_{ij} \times M_j$ in section I.A) is expressed in dollars per unit, while the SSNIP used in market definition tests is expressed as a percentage price increase. Expressing the GUPPI as a percentage of price makes the connection to market definition clear.

21 Salop and Moresi assumed equal prices for the two merging firms. Salop & Moresi, supra note 4.
Like the UPP test, the GUPPI in and of itself is a useful screen and might be used as supporting “circumstantial” evidence, but it also is not a complete analysis of all the relevant competitive factors.

**B. Relationship to Market Definition.** The main advantage of the GUPPI over the UPP is that the GUPPI can be directly related to market definition. As a result, the GUPPI approach might be more attractive to antitrust practitioners and the courts, where market definition is a key component of antitrust analysis.

In particular, the GUPPI is closely related to the market definition methodology developed by Katz & Shapiro and O’Brien & Wickelgren.22 Like the GUPPI, this methodology is based on the standard model of Bertrand competition with differentiated products. Specifically, the products of the two merging firms would comprise a relevant antitrust market if:

\[ \text{GUPPI}_1 \geq 2s \text{ (or if GUPPI}_2 \geq 2s) \]

where \( s \) denotes the level of “small but significant non-transitory increase in price” (SSNIP) used for market definition.23 The two products would comprise a relevant market because, if the above condition is satisfied, then a hypothetical monopolist who would be the sole owner of the two products would find it profit-maximizing to raise the price of Product 1 alone (or Product 2 alone) by at least a SSNIP, even if one assumed that it did not also raise the price of the other product. According to the current Guidelines, this would be a sufficient condition for the two products to constitute a relevant market.

For example, if the relevant SSNIP is \( s = 5\% \), then the products of the two merging firms alone would comprise a relevant market if the GUPPI were at least 10\% (i.e., \( 2 \times 5\% \)) for one of the products of the merging firm.24 If so, the relevant market would include only the products of the merging firms. Absent production substitution or other uncommitted entry, the merger could be characterized as a “merger to monopoly.”

It is noteworthy that the GUPPI demonstrates that there can be a relevant market composed of a subset of products that are equally close substitutes. Consider a hypothetical broad market with six equal-size firms, each producing a single product that is an equally close substitute for each of the other five products. Assume for simplicity that these products have equal prices and equal margins, and that total demand is perfectly inelastic. In this case, the diversion ratio between any

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23 This formula assumes that market definition is based on a profit-maximizing SSNIP, not a just-profitable (break-even) SSNIP. In addition, like the UPP test, it assumes that each of the merging firms produces a single product, and that other firms would not change the prices of their products. Unlike the UPP test, the GUPPI test further assumes that the merging firms face linear demand and constant marginal cost. These last assumptions are not necessary for the UPP test, but are necessary to illustrate the link between GUPPI and market definition. (Katz & Shapiro assume linear demand, while O’Brien & Wickelgren assume constant elasticity demand. In contrast, the UPP test does not make any functional form assumptions about demand.)

24 The formula assumes a single-product SSNIP—that is, the hypothetical monopolist would raise the price of only one of the two merging products. Therefore, the test in the text offers a sufficient but not necessary condition. That is, the two merging products may constitute a relevant market even if the two GUPPIs are smaller than 10 percent. If instead one assumes a uniform SSNIP—that is, the hypothetical monopolist would raise the prices of the two merging products by identical SSNIPs—then a simple formula can be obtained only if one assumes symmetric merging products (i.e., equal shares, equal prices, equal margins, and equal diversion ratios). See Salop & Moresi, *supra* note 4, at 21 n.48. Note that the symmetry assumption is unlikely to be satisfied in most cases.
two firms would be 20 percent. If the margin were higher than 50 percent, then the GUPPI would exceed 10 percent and, therefore, any two firms in this industry would constitute a relevant antitrust market. If instead the industry is comprised of only three firms (instead of six), the diversion ratio between any two firms would be 50 percent. In this case, any two firms also would constitute a relevant market if the profit margin was greater than 20 percent because that would imply a GUPPI above 10 percent.

C. Presumptions and Safe Harbors. The GUPPI might be used for setting enforcement thresholds for unilateral effects concerns in differentiated products industries, based on its relationship to market definition (as well as other considerations, as discussed below). The GUPPI thus could be used to replace or supplement the current thresholds based on the HHI and the combined market share of the merging firms. For example, a relatively low GUPPI threshold could be used to establish a "safe harbor" for unilateral effects concerns, either as an initial screen or later on in the merger review process. The Agencies (or the courts) also could use the GUPPI to determine whether or not to "presume" harmful unilateral effects and shift the burden of rebuttal onto the merging parties. Most economists probably would agree that the GUPPI generally is a better gauge of unilateral effects concerns than the product of the shares (i.e., the HHI delta) or the combined market share of the merging firms.

If the GUPPI were given this role, the Agencies would need to set the thresholds and the strength of the safe harbor and anticompetitive presumptions. As suggested by the examples above, the Agencies might consider using a GUPPI threshold of 10 percent, since this characterizes what could be deemed to be a “merger to monopoly,” absent production substitution or other uncommitted entry. Of course, other thresholds could be used.

Alternatively, the Agencies could consider using the UPP test with a relatively high “efficiency credit” to identify presumptively anticompetitive deals, while using a lower credit to identify non-harmful deals that might fall in the safe harbor. However, with the UPP, it would be more difficult to relate these presumptions to market definition and thus justify the efficiency credit thresholds that the Agencies would be using. The GUPPI approach therefore might be seen as more practical.

Conclusion

The UPP test for unilateral effects is based on the assumption of Bertrand competition. This paper explains how the UPP test can be extended to other forms of competition. In particular, the UPP test can be applied to merger cases where firms engage in Cournot competition and also to cases with bidding competition. In each case, the paper explains how to define and calculate the rele-

25 If a firm raises price unilaterally and loses 100 customers, then those 100 customers all stay within the industry (since demand is perfectly inelastic) and thus each one of the 5 rivals of that firm gains 20 customers. It follows that the diversion ratio between any two firms is 20 percent. When demand is not perfectly inelastic, the diversion ratio should take into account the substitution to “outside goods” expressed in the demand elasticity.

26 This also likely would take into account deterrence concerns. See Salop & Moresi, supra note 4.

27 In addition, using the UPP test (instead of the GUPPI) for establishing presumptions would raise the issue of whether mergers in low-cost industries would be more likely to fail the test all else equal. This issue could be addressed by choosing industry-specific efficiency credits. A better approach could be to express the efficiency credit as a percentage of price (not cost).
vant “diversion ratio” to be used in the UPP test. This suggests that the UPP test can be used in most merger cases.\textsuperscript{28}

The paper also points to several factors that are not accounted for by the UPP test, but are important for a complete evaluation of the potential unilateral effects. This suggests that the UPP test (and the GUPPI) should be used as an initial screen or in conjunction with other factors currently used in merger analysis, not as a complete analysis of competitive effects.

Unlike the UPP test, the GUPPI test does not attempt to account for efficiencies. But unlike the UPP test, the GUPPI test can be related directly to the hypothetical monopolist market definition test. This might make the GUPPI a more intuitive index for antitrust practitioners and the courts. The article explains how the Agencies could use this relationship between the GUPPI and market definition to set GUPPI thresholds and establish safe harbors or anticompetitive presumptions. Of course, the Agencies could decide to set the GUPPI thresholds based on several factors other than simply their relationship to market definition. In particular, the thresholds might be adjusted to account for “optimal deterrence” considerations or reflect a particular “enforcement philosophy.” Whatever the exact GUPPI thresholds that are chosen, this basic approach is both useful and defensible.

\textsuperscript{28} In some industries, firms sometime compete through “tournaments” or “beauty contests” by investing in the quality of the product offered to the customer (e.g., architects competing for a $100 million construction contract). In other industries, firms sometime compete in “bargaining markets” (e.g., each major hospital signs a contract with each major HMO). It is unclear whether the UPP test can be extended and applied to these types of industry structures.
This appendix describes the derivations of the formulas presented in this article. Section A.1 presents a unified framework and derives a general result that includes the UPP and GUPPI tests as special cases. Specifically, it develops a general test of the profitability of a single-product SSNIP post-merger in the presence of cost savings. Section A.2 then explains how this general test can be used to derive the UPP and GUPPI tests. In addition, the general test also can be used to derive the post-merger profit-maximizing single-product SSNIP.

A.1 A unified framework for single-product Price Pressure Indices

Consider a hypothetical merger of Firm 1 and Firm 2. Firm 1 sells Product 1, Firm 2 sells Product 2, and the pre-merger prices of Product 1 and Product 2 are denoted by $P_1$ and $P_2$, respectively. Following Farrell & Shapiro, I analyze the merged firm’s incentive to raise $P_1$, assuming for simplicity that (i) all the other prices (including $P_2$) will remain constant at their pre-merger level, and (ii) the merger will not generate any variable cost savings for Product 2.29

If the merged firm does not change the prices of Product 1 and Product 2, then its total profits are equal to:

$$ (P_1 - C_1 + E_1)Q_1 + (P_2 - C_2)Q_2 $$

where $C_1$ and $C_2$ are the marginal costs of production of Product 1 and Product 2, $Q_1$ and $Q_2$ are the sales volumes of Product 1 and Product 2, and $E_1$ is the reduction in the marginal cost of Product 1 generated by the merger.

If instead the merged firm unilaterally raises the price of Product 1 from $P_1$ to $(1 + S)P_1$, then its total profits are equal to:

$$ ((1 + S)P_1 - C_1 + E_1)(1 - L)Q_1 + (P_2 - C_2)(Q_2 + DR_{12}LQ_1) $$

Thus, the increase in the price of Product 1 leads to a reduction in the sales volume of Product 1 from $Q_1$ to $(1 - L)Q_1$, where $L$ is the “loss” of volume expressed as a fraction of the initial (pre-merger) volume of Product 1. At the same time, the increase in the price of Product 1 leads to an increase in the sales volume of Product 2 from $Q_2$ to $Q_2 + DR_{12}LQ_1$, where $DR_{12}$ is the diversion ratio from Product 1 to Product 2.30

By comparing (1) and (2), one can analyze the merged firm’s incentive to change the price of Product 1 from $P_1$ to $(1 + S)P_1$.31 Let $m_1$ and $m_2$ denote the profit margins of Product 1 and Product 2, i.e., $m_1 = (P_1 - C_1)/P_1$ and $m_2 = (P_2 - C_2)/P_2$.

29 As explained in Farrell & Shapiro, these assumptions tend to underestimate the merged firm’s incentive to raise the price of Product 1. See Farrell & Shapiro, Economic Alternative to Market Definition, supra note 1, §§ 2.C and 2.D; see also Farrell & Shapiro, UPP and Critical Loss Analysis, supra note 1, §§ III and IV.

30 $LQ_1$ is the volume lost by Product 1 as a result of the unilateral increase in the price of Product 1, $DR_{12}$ is the fraction of that lost volume that is recaptured by Product 2, and hence $DR_{12}LQ_1$ is the amount of volume gained by Product 2 as a result of that price increase.

31 The analysis also applies to a price reduction from $P_1$ to $(1 + S)P_1$, where $S$ is negative.
Proposition. Assume that pre-merger prices are Bertrand equilibrium prices. If either demand is linear or the price change is very small (i.e., infinitesimal), then (2) is greater than (1) if and only if

\[ DR_{12} m_2 P_2 > E_1 + SP_1 \]  (3)

Proof. Subtracting (1) from (2), one finds that (2) > (1) if and only if

\[ SPQ_1 - (P_1 - C_1 + SP_1 + E_1)LQ_1 + DR_{12}(P_2 - C_2)LQ_1 > 0 \]  (4)

Using \( m_kP_k = P_k - C_k \) to substitute for \( P_k - C_k \) (where \( k = 1,2 \)), and dividing throughout by \( SQ_1 \), one finds that (4) is equivalent to:

\[ P_1 - (m_1P_1 + SP_1 + E_1)(L/S) + DR_{12} m_2 P_2 (L/S) > 0 \]  (5)

Note that \( L/S \) is the own-price elasticity of the demand for Product 1 (evaluated at the pre-merger equilibrium prices) if either \( S \) is infinitesimally small or if demand is linear. Thus, pre-merger Bertrand equilibrium implies \( m_1 = S/L \). (This is called the “Lerner condition.”) Thus, multiplying (5) by \( m_1 \), one obtains:

\[ m_1P_1 - (m_1P_1 + SP_1 + E_1) + DR_{12} m_2 P_2 > 0 \]  (6)

Then, (3) is obtained by collecting and rearranging terms. ●

The next section shows how the above Proposition can be used to derive the UPP test, the GUPPI test, and the post-merger profit-maximizing single-product SSNIP.

A.2 Applications

The UPP test. The UPP test assumes that there are significant efficiencies, i.e., \( E_1 > 0 \), and asks whether the merged firm would have an incentive to raise \( P_1 \) by even an infinitesimally small amount, i.e., \( S = 0 \). From (3), the answer is affirmative if \( DR_{12} m_2 P_2 > E_1 \). Since \( m_2 P_2 = M_2 \), the UPP test can be rewritten as in section I.A, i.e., \( DR_{12} \times M_2 > E_1 \).

The GUPPI test. The single-product SSNIP test assumes that there are no efficiencies, i.e., \( E_1 = 0 \), and asks whether a hypothetical monopolist (who would be the sole owner of Product 1 and Product 2) would have an incentive to raise \( P_1 \) by at least a SSNIP, i.e., \( S > 0 \). As I will explain in the next paragraph, one should use \( S = 2s \), where \( s \) is the level of the SSNIP used for market definition. From (3), the answer is affirmative (and thus the two products would constitute a relevant antitrust market) if \( DR_{12} m_2 P_2 / P_1 > 2s \). This can be rewritten as in section II.B, i.e., \( GUPPI \geq 2s \).

Suppose it were true that \( S = 2s \) implies (2) = (1), and thus (3) would hold with equality. In this case, the hypothetical monopolist would be indifferent between not raising the price of Product 1 (i.e., \( S = 0 \)) and raising it from \( P_1 \) to \( (1 + 2s)P_1 \) (i.e., \( S = 2s \)). Thus, if the hypothetical monopolist were to raise price by \( S = 2s \), it would “break even” relative to no price increase. Since demand is assumed to be linear, if the break-even price increase is equal to \( 2s \), then the profit-maximizing price increase is equal to \( s \). Therefore, since the current Guidelines instruct us to use a profit-maximizing SSNIP (as opposed to a just profitable SSNIP) and since condition (3) is a break-even test, we should use \( S = 2s \) when implementing the GUPPI test.

The post-merger profit-maximizing single-product SSNIP. Assuming that (3) holds with equality and solving for \( S \) yields:
\[ S^* = (DR_{12} m_2 P_2 - E_1) / P_1 \]  

(7)

This is the “break-even” or “just profitable” single-product SSNIP for the merged firm. Thus, the “profit-maximizing” single-product SSNIP that the merged firm would impose on Product 1 is given by:

\[ s^* = (GUPPI_1 - e_1) / 2 \]  

(8)

where \( e_1 = E_1 / P_1 \) is the cost savings “efficiency credit” expressed as a percentage of price (not cost). It follows that the GUPPI test and the UPP test are equivalent if the efficiency credit used in the UPP test is expressed as a percentage of price (as was suggested supra note 27).