Improving Critical Loss Analysis

Joseph Farrell and Carl Shapiro

Market definition analysis usually claims to follow the 1992 Horizontal Merger Guidelines issued by the U.S. Department of Justice and the Federal Trade Commission.[1] The Merger Guidelines describe a market as a group of products for which a hypothetical monopolist would profitably impose a small but significant and non-transitory increase in price (SSNIP). Seeking relatively simple approaches to market definition that are consistent with the Merger Guidelines, courts and agencies often rely on Critical Loss Analysis.[2]

For example, the FTC recently challenged the proposed merger between Whole Foods and Wild Oats, two chains of grocery stores,[3] alleging that the relevant market was "premium natural/organic supermarkets" (PNOS). In that market, the merger was very highly concentrating in a number of geographic areas where Whole Foods and Wild Oats operated nearby stores. But the merging parties successfully argued that PNOS was too narrow a grouping of products and that the market included all supermarkets. In that broader market, there were many other competitors and the merger was not highly concentrating.

Arguing that PNOS was not a market, the merging firms echoed the Merger Guidelines by asking whether a hypothetical PNOS monopolist would find a SSNIP profitable or whether a SSNIP would deter enough sales to make it unprofitable. Critical Loss Analysis calculates the hypothetical monopolist's Critical Loss, meaning the magnitude of lost sales that would (just) make it unprofitable for the hypothetical monopolist to impose a SSNIP, and compares it against the so-called Actual Loss of sales that would result from the SSNIP. If the Actual Loss would be less than the Critical Loss, the SSNIP would be profitable, so PNOS would form a market. Whole Foods and Wild Oats argued that the Actual Loss would instead exceed the Critical Loss: A hypothetical PNOS monopolist that imposed a SSNIP would lose enough business to make the SSNIP unprofitable. Merging parties have used Critical Loss Analysis regularly, and with considerable success, to argue in court for a broader market than the government asserts.[4]

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[1] U.S. Dep't of Justice & Fed. Trade Comm'n, Horizontal Merger Guidelines (1992, revised 1997), http://www.ftc.gov/bc/docs/horizmer.htm. Distinguished antitrust economist and lawyer Jonathan Baker begins a recent article by noting: "Throughout the history of U.S. antitrust litigation, the outcome of more cases has surely turned on market definition than on any other substantive issue." Jonathan Baker, *Market Definition: An Analytical Overview*, 74 ANTITRUST L.J. 129, 129 (2007).

[2] The term "Critical Loss" was introduced by Barry Harris and Joseph Simons in *Focusing Market Definition: How Much Substitution Is Necessary?* 12 RES. L. & ECON. 207 (1989).

[3] FTC v. Whole Foods Mkt., Inc., 502 F. Supp. 2d 1 (D.D.C. 2007). For a discussion of Whole Foods expressing skepticism about the FTC's case, see Carlton Varner & Heather Cooper, *Product Markets in Merger Cases: The* Whole Foods *Decision*, ANTITRUST SOURCE, Oct. 2007, http://www.abanet.org/antitrust/at-source/07/10/Oct07-Varner.pdf.

[4] Michael Katz, *Recent Antitrust Enforcement Actions by the U.S. Department of Justice: A Selective Survey of Economic Issues*, 21 REV. INDUS. ORG. 373 (2002) (explaining how Critical Loss Analysis was used by the merging parties in *United States v. SunGard Data Systems*, Inc., 172 F. Supp. 2d 172 (2001)). Additional cases are described in Daniel O'Brien & Abraham Wickelgren, *A Critical Analysis of Critical Loss Analysis*, 71 ANTITRUST L.J. 161 (2003). *See also* Barry Harris & Cento Veljanovski, *Critical Loss Analysis: Its Growing Use in Competition Law*, 5 EUROPEAN COMPETITION L.J. 213 (2003).

Estimating a hypothetical monopolist's Actual Loss is difficult, so that a substantial range of estimates could seem plausible. Incentives in litigation may push parties toward exploiting that range. Thus, it is highly desirable, if possible, to anchor estimates of Actual Loss and to facilitate reality checks based on actual premerger conduct. When it comes to demand responsiveness, economics suggests that it is particularly helpful to examine firms' own premerger pricing conduct. As discussed below, some economists have explored how to exploit such evidence, but others have suggested that premerger pricing is sufficiently remote from the hypothetical monopolist question that these reality checks are unhelpful. In this article we explore more deeply how some of the complexities of premerger pricing might be thought to undermine the link with the demand elasticity faced by the hypothetical monopolist. We find that premerger pricing, together with estimates of diversion ratios, can be highly informative about a hypothetical monopolist's incentives.

Merging parties have used Critical Loss Analysis regularly, and with considerable success, to argue in court for a broader market than the government asserts.

We can thus offer two powerful new tests to determine, using Critical Loss Analysis, whether a candidate group of products contains enough substitutes to form a market.[5] These tests extract information from the gold standard for evidence about competitive conditions in antitrust cases: firms' decisions actually made in the normal course of business.

1. A Short Refresher Course in Critical Loss Analysis

Suppose that a group of products, such as PNOS, is proposed as a candidate market. How would one know, following the Merger Guidelines, whether a hypothetical monopolist would find it profitable to impose a SSNIP? Simple economics tells us that the answer is, "It would be profitable if demand is not too elastic." But that answer alone is not very helpful.

Critical Loss Analysis contributes by asking quantitatively just *how* elastic demand must not be, for the candidate market to be an antitrust market. This part of the analysis yields a number for the Critical Loss. It is often described as "just arithmetic," although, as we shall see, that is a simplification. The meat of the analysis then involves estimating the Actual Loss—by no means a simple task. We may have no recent experience with a monopolist over just that group of products, and typically no business plans will exist for such a hypothetical monopoly. On the consumer side, while marketing studies in the ordinary course of business may well ask customers, "If product X were unavailable, or more expensive, what would you buy instead?" they seldom ask, "If products X, Y, and Z were unavailable, or more expensive, what would you buy?"[6]

In practice, Critical Loss Analysis typically assumes that the products are symmetric in price and cost, and studies only a uniform SSNIP imposed on all products.[7] The assumption of product symmetry will, of course, seldom be correct, and a hypothetical monopolist might well want to raise some prices much more than others. In our companion paper we develop an alternative technique that is better suited to analyzing product asymmetries or non-uniform price increases.[8]

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[5] In this article we follow the Merger Guidelines methodology largely without question. In an ongoing work, Mergers with Unilateral Effects: A Simpler and More Accurate Alternative to Market Definition, we criticize the market definition approach and offer an alternative in cases involving unilateral effects. This paper will be available at http://faculty.haas.berkeley.edu/shapiro.

[6] Indeed, Harris and Veljanovski suggest that estimation of Actual Loss is apt to be too difficult, so that one should instead only gauge the plausibility of its being below the calculated Critical Loss. *See* Harris & Veljanovski, *supra* note 4, at 215.

[7] The asymmetric case can be analyzed, with additional complexity. *See, e.g.*, Serge X. Moresi, Steven C. Salop & John R. Woodbury, *Implementing the Hypothetical Monopolist SSNIP Test with Multi-Product Firms*, Antitrust Source, Feb. 2008, http://www.abanet.org/antitrust/at-source/08/02/Feb08-Moresi.pdf.

[8] Joseph Farrell & Carl Shapiro, Cannibalization, Pass-Through, and Market Definition, uses a different technique and provides additional tests for market definition, also following the Guidelines, that allow for asymmetry among the products in the candidate market. This paper will be available at author Carl Shapiro's Web site, http://faculty.haas.berkeley.edu/shapiro/.

Here we adopt the conventional simplification of symmetric products and a uniform SSNIP. We also make the standard assumption in Critical Loss Analysis that the hypothetical monopolist produces at constant marginal cost.

Let the premerger price be p, and the constant marginal cost be c, so the premerger gross margin is $m = (p - c)/p$. Let the size of the SSNIP (as a percentage of premerger price) be s; following the Guidelines, we often illustrate with a 5 percent SSNIP, so $s = .05$ (that is, 5%). Let the premerger unit sales in the candidate market be X.

A. Critical Loss Arithmetic and Demand Sensitivity to Price Increase. Critical Loss analysis begins by calculating the Critical Loss, i.e., the (proportional) loss of sales that would just make a SSNIP unprofitable. In this calculation, the hypothetical monopolist's profit function is normally simplified to include only the direct, immediate, and quantifiable costs and revenues, marginal cost is simplified to be constant, and the SSNIP is simplified to be uniform. Using this simple proxy, the Critical Loss measured as a fraction of the hypothetical monopolist's premerger sales is given by $s/(m + s)$.[9] For example, with a margin of 45 percent and a SSNIP of 5 percent, the Critical Loss is 5/(45 + 5) = 1/10, or 10 percent. The SSNIP will be profitable if and only if the actual loss of sales, as a fraction of premerger sales, is less than the Critical Loss.

This calculation asks: "Would a hypothetical monopolist find a SSNIP more or less profitable than the status quo?" This "break-even" version of the hypothetical monopolist's pricing incentives fits naturally with the separation of Critical Loss and Actual Loss calculations, and the approach has been accepted by a number of courts, including the *Whole Foods* court, so we pursue it here through Section 5 below. The Merger Guidelines actually ask the related but distinct question: "Is the hypothetical monopolist's profit-maximizing price at least a SSNIP above the status-quo level?"[10] Section 6 discusses the relationship between the breakeven and profit-maximizing approaches, and shows in the case of linear demand how to modify our analysis to handle the profit-maximizing approach.

B. Estimating Actual Loss: The Role of Premerger Margins. The second step in Critical Loss analysis is to estimate the number of lost sales, Z, or equivalently the fraction of premerger sales, Z/X, that will be lost due to a SSNIP. We define the Actual Loss as this fraction: $L = Z/X$.[11] Estimating the Actual Loss requires evidence about buyer substitution patterns. The controversy over Critical Loss Analysis concerns where to look for such evidence—specifically, how much can be inferred from premerger margins.

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[9] All technical derivations are in the Appendix.

[10] Section 1.0 of the Merger Guidelines defines a market as "a product or group of products and a geographic area in which it is produced or sold such that a hypothetical *profit-maximizing* firm, not subject to price regulation, that was the only present and future producer or seller of those products in that area likely would impose at least a 'small but significant and nontransitory' increase in price, assuming the terms of sale of all other products are held constant. A relevant market is a group of products and a geographic area that is no bigger than necessary to satisfy this test." (Emphasis added.) Section 1.11 states that "In performing successive iterations of the price increase test, the hypothetical monopolist will be assumed to pursue *maximum* profits in deciding whether to raise the prices of any or all of the additional products under its control." (emphasis added) One economist actively involved in the development of the Merger Guidelines reports that profit maximization was intended even in the 1982 Guidelines and that the 1984 revision included a revision "in an attempt to make [this] clear." Gregory J. Werden, *The 1982 Merger Guidelines and the Ascent of the Hypothetical Monopolist Paradigm*, 71 ANTITRUST L.J. 253, 258 (2003).

[11] Confusingly, the term "Actual Loss" means the *predicted* loss that would result from a *hypothetical* monopolist's SSNIP. Unfortunately, the term has become standard.

Katz and Shapiro[12] and O'Brien and Wickelgren[13] have shown how to estimate Actual Loss using information about demand responsiveness based on firms' premerger pricing decisions. This is what economists call "revealed preference" information: inferences about preferences based directly on observed choices. Here, one can make inferences about demand sensitivity, as gauged by a real firm based on its premerger choice of price. In particular, if (before the merger) a firm chooses a high margin on its product, the firm evidently thinks that demand for its product is not very sensitive to price.

This idea is captured in the Lerner Equation, the textbook centerpiece of pricing economics. The Lerner equation states that, if a product is priced so as to maximize the profits from that product, then the proportional gross margin, m, will be the inverse of the elasticity of demand facing the product ϵ: that is, $\epsilon = 1/m$.[14] For example, a margin of 50 percent (one-half) will be chosen when the firm faces an elasticity of two.

Can one use the Lerner Equation to arrive at a reliable estimate of the hypothetical monopolist's Actual Loss in Critical Loss Analysis?

The Lerner Equation directly implies a prediction of the sales that would be lost by a single product if its price were to rise by a SSNIP. By the definition of elasticity, the fraction of sales lost is approximately ϵs,[15] and by the Lerner Equation, that in turn equals s/m. With a profit-maximizing margin of 45 percent, a single product whose price rises by a 5 percent SSNIP can be expected to lose a fraction 5/45, or 11.1 percent, of its sales.

Can one use the Lerner Equation to arrive at a reliable estimate of the hypothetical monopolist's Actual Loss in Critical Loss Analysis?[16] In stark form, the contending positions are:

- *Stress Premerger Pricing Evidence.* If we want to know how the sales of a product, or a group of products, would respond to a price change, the best evidence normally is what the firms owning those products think, and what they do when their money is on the line. These firms know the market for their products better than any outside expert or court is likely to, and decisions taken in the normal course of business are less likely to be biased than is analysis developed in an adversarial setting. Economists generally treat revealed-preference information, i.e., direct inferences from actual choices, with special respect. Economics can fairly reliably infer a firm's beliefs about its demand elasticity from the firm's premerger behavior, specifically, the price that it chose for its product. Other information on demand responsiveness that was known to the firm should already be factored into its choice of price, and thus should be given little or no separate weight. Information on demand responsiveness that was available to the firm, but that the firm chose not to use, is probably not all that valuable, as judged by

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[12] Michael L. Katz & Carl Shapiro, *Critical Loss: Let's Tell the Whole Story*, ANTITRUST, Spring 2003, at 49.

[13] O'Brien & Wickelgren, *supra* note 4.

[14] A modified version of the Lerner Equation applies if the price affects the firm's profits in ways not captured by the margins the firm earns on this product alone. *See infra* Section 4.

[15] The Appendix shows that this approximation is exact (in the way we use it) when demand is linear. Of course, demand will never be exactly linear, but for small changes in price, departures from linearity should not normally cause large errors in calculating the change in profits. We thus view the linear calculations as a reasonable first approximation, which is all that market definition seeks in any case; if it suggests a close call, then one can look for curvature or kinks, as we discuss in Section 3 below.

[16] This approach to Critical Loss Analysis was explored by Michael Katz and Carl Shapiro, *supra* note 12, and by Daniel O'Brien and Abraham Wickelgren, *supra* note 4. Arguments against relying on the Lerner Equation were presented by David Scheffman and Joseph Simons in *The State of Critical Loss Analysis: Let's Make Sure We Understand the Whole Story*, ANTITRUST SOURCE, Nov. 2003, http://www.abanet.org/antitrust/at-source/03/11/scheffman.pdf. For responses to Scheffman and Simons, see Michael Katz & Carl Shapiro, *Further Thoughts on Critical Loss*, ANTITRUST SOURCE, Mar. 2004, http://www.abanet.org/antitrust/at-source/04/03/katzshapiro.pdf, and Daniel O'Brien & Abraham Wickelgren, *The State of Critical Loss Analysis: Reply to Scheffman and Simons*, ANTITRUST SOURCE, Mar. 2004, http://www.abanet.org/antitrust/at-source/04/03/obrienwickel.pdf.

the firm when it had money on the line. Thus, a good, simple way to estimate Actual Loss is to extrapolate, using economic logic, from the firms' premerger pricing choices.

- *Ignore Premerger Pricing Evidence.* The use of revealed-preference information in this context is badly flawed for several reasons. Firms do not always maximize profits. Even if they do, maximizing profits in the real world is very complex, often requiring the firm to take into account spillovers to its other (substitute or complement) products, but also involving customer loyalty, reputation, learning-curve effects, network effects, and so on. These factors are not included in any practicably implementable form of the Lerner Equation. Moreover, in an oligopoly, other firms may respond to any one firm's price changes, so a rational firm will not set its price to maximize profits taking as given its rivals' prices. Indeed, some economists have suggested that this often yields a "kink" in the demand curve facing a firm once rival responses are taken into account. Thus, in estimating the Actual Loss, it is better to give little or no weight to the firm's premerger choice of price and, instead, to seek direct evidence about demand responsiveness. Such evidence can include econometric or marketing studies and/or an intuitive evaluation of qualitative facts about what it would take for a customer to substitute away from a group of products. If the answer appears at odds with firms' premerger prices and margins, one should not be unduly puzzled or concerned.

In Sections 2 to 4 we address the concerns raised in the second bullet about the use of premerger margins to estimate the Actual Loss. To make the concept concrete, however, we begin by reviewing how premerger margins can be used in the simplest case.

How Premerger Choices Reveal Demand Sensitivity: The Simple Case. Katz and Shapiro, and O'Brien and Wickelgren, develop a simple test for market definition using two ingredients. The first ingredient is the premerger margin, m. The second is the Aggregate Diversion Ratio, A, which is the fraction of the sales lost by one product, when its price alone rises by a SSNIP, that go to other products in the candidate market. Equivalently, one can think of $(1 - A)$ as the fraction of a product's demand elasticity that consists of substitution to goods *outside* the candidate market.

Thus the elasticity of demand facing the hypothetical monopolist is only a fraction $(1 - A)$ of that facing any one product. This directly implies an estimate of Actual Loss of $L = (1 - A)\epsilon s$; this estimate is precise if demand is linear for small price changes. So, if we can use the Lerner Equation, the Actual Loss equals $L = (1 - A)(s/m)$. For example, with a SSNIP of 5 percent, a margin of 45 percent, and an Aggregate Diversion Ratio of 25 percent, the Actual Loss is estimated to be 8.3 percent, while the Critical Loss is 10 percent, so the group of products would form a market. Following this logic, in the symmetric case, gives:[17]

> *Proposition 1 (Katz-Shapiro-O'Brien-Wickelgren):* If each firm owns a single product and prices to maximize its profits taking as given all other prices, and if demand for each product is linear in price for small price changes from the premerger price, then a symmetric group of products forms a market under break-even analysis if and only if
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> $$A \geq \frac{s}{m+s}.$$

[17] This is the test for whether a uniform SSNIP, imposed on all the (symmetric) products in the candidate market, is more profitable than the status quo price. A different condition diagnoses whether the hypothetical monopolist would find a SSNIP imposed on just one product more profitable than the status quo. As Øystein Daljord, Lars Sørgard & Øyvind Thomassen note in *The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro*, J. Competition L. & Econ. (forthcoming 2008), this latter condition is that $A \geq s/m$. Because this latter condition is less easily satisfied than the one in Proposition 1, Proposition 1 accurately follows the Merger Guidelines test in the symmetric case. As Daljord et al. stress, the asymmetric test follows the Merger Guidelines in asymmetric situations where an asymmetric SSNIP is more apt to be profitable than is a symmetric one.

We do not suggest that this revealed preference approach is the only way to estimate Actual Loss. Nor should it be ignored, however. If one had a reasonably reliable estimate of m and of the Aggregate Diversion Ratio A, one should be skeptical of a separate estimate of Actual Loss that departed much from the predicted value of $L = (1 - A)(s/m)$. In our numerical example, a separate estimate of Actual Loss much greater than 8.3 percent would require reconciliation with the estimate of 25 percent for the Aggregate Diversion Ratio. We discuss how to handle such conflicting evidence in Section 5 below.

Evidence on the premerger margin can come directly from measurements of the premerger price and marginal cost. While there may be some debate about the level of marginal cost, there is often good information in company documents about marginal cost (or some appropriate version of average incremental cost) and one can always perform sensitivity analysis on the level of marginal cost.

Evidence on the Aggregate Diversion Ratio may be gleaned from surveys of customer switching patterns or past customer responses to changes in prices or product availability. Econometric evidence based on demand responses to price changes also can help in measuring A. As usual, one must seek to avoid inadvertently measuring price responses to demand shifts. Recall that A is calculated on the assumption that the price of one product changes and the other prices do not.

Proposition 1 implies that a seemingly narrow group of products will often form a market according to the Guidelines. With the standard SSNIP of $s = .05$ and with a moderate margin of $m = .45$, a group of products forms a market if $A \geq 0.1$. In many intuitively defined "industries," the Aggregate Diversion Ratio would be far higher, so narrower markets may well exist within the industry. For instance, if the price of one model, or brand, of cars were to rise by a SSNIP, quite a few customers would no doubt substitute away—but we would expect that most of them would substitute away to some other car. Thus, if gross margins are about 45 percent, there would be a product market considerably narrower than "cars." For example, if 20 percent of BMW customers would substitute to Mercedes or Audi following a SSNIP by BMW, and conversely, then the hypothetical monopolist test suggests that "German luxury cars" would be a market.[18] Certainly this arithmetic suggests that "cars" would easily be a market, and the hypothetical monopolist methodology would not imply that pickup trucks or minivans would need to be included. Whether the relatively narrow groups of products that often emerge from the Merger Guidelines methodology are the most informative way to define a market is a deep issue beyond the scope of this paper.

The *Whole Foods* court appeared to believe that a group of products could not form a market if the Aggregate Diversion Ratio is significantly below one-half, but our analysis shows that this view is inconsistent with the Merger Guidelines if, as is normally the case, m substantially exceeds s.[19]

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[18] In this article, we focus on whether a SSNIP would be profitable (either more profitable than the status quo, or profit maximizing), not on whether the candidate market contains the closest substitutes, in order, starting from one product of one of the merging firms.

[19] The court stated: "In other words, when a Whole Foods enters an area that has a Wild Oats store, its sales do not overwhelmingly come from Wild Oats, but primarily from other supermarkets; the main competitive interaction is between Whole Foods and 'other' grocery retailers." FTC v. Whole Foods Mkt., Inc., 502 F. Supp. 2d 1, 20 (D.D.C. 2007). While the public information on *Whole Foods* does not appear to include an estimate of m, there is evidence that the margin was far larger than the size of a SSNIP. The public version of the rebuttal report of the FTC's expert Dr. Kevin Murphy indicates that even with an aggregate diversion ratio of only 0.1 to 0.2, premium/natural organic supermarkets would constitute a relevant market according to a test equivalent to Proposition 1. *See* Expert Report of Kevin M. Murphy ¶ 28, FTC v. Whole Foods Mkt., Inc., 502 F. Supp. 2d 1 (D.D.C. 2007) (Civ. No. 07-cv-01021-PLF), *available at* http://www.ftc.gov/os/caselist/0710114/070823murphy.pdf. SSNIP sizes of 5% and 1% were discussed earlier in his report. Using a SSNIP of 5% and an aggregate diversion ratio of 20% would, according to Proposition 1, suggest a gross margin of 20%, four times the SSNIP. Using a SSNIP of 1% and an aggregate diversion ratio of 10% would suggest a gross margin of 9%, nine times the SSNIP.

But what if the assumptions behind Proposition 1 are not accurate? In Section 2 we generalize Proposition 1 to cover industries in which the premerger firms respond to one another's price changes. In Section 3 we generalize Proposition 1 to allow for the possibility that demand is not linear in price—in particular that demand becomes more sensitive to price increases as the price rises above the premerger level. In Section 4 we discuss how Critical Loss Analysis can be useful even when firms' real-world profit functions include various subtle and complex factors.[20]

2. Accounting for Pricing Responses by Rivals

Much quantitative merger analysis assumes in practice that each firm sets its price taking as given the other firms' prices. So far, we have followed that approach. However, this may not accurately reflect firms' actual behavior in many oligopolies in which mergers would be challenged these days. Indeed, whether a firm tracks, and responds to, other firms' price changes is often taken as an indicator of whether they compete or are in the same market.[21]

If premerger oligopolists are in the habit of responding to one another's price changes, then a firm that contemplates a price change will not care how its sales *would* respond if other prices were held fixed, but will seek to predict how its sales *will* respond once others react to that change as they are expected to do in the real-world oligopoly.[22] In general, then, revealed-preference information illuminates price sensitivity on this *residual demand curve*, not (directly) on the conventional demand curve that holds other prices fixed.[23]

Again, some fraction of the sales lost by any one product through its SSNIP go to other products in the candidate market, but now this fraction must be measured using the residual demand curve, i.e., accounting for the price responses of the other products. We call this the Residual Aggregate Diversion Ratio, or A^*. The Appendix proves:[24]

> *Proposition 2:* If each firm owns a single product and prices to maximize its profits accounting for price responses by other firms in the candidate market, and if demand for each product is linear in price for small price changes starting from the premerger price, then a symmetric group of products forms a market under break-even analysis if
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> $$A^* \geq \frac{s}{m+s},$$
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> but may do so even if that condition fails to hold.

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[20] In a few places we combine these analyses, but that is largely left for future work: we primarily relax the simplifying assumptions one at a time.

[21] For example, the *Whole Foods* court noted that Whole Foods tracks prices in mainstream supermarkets. 502 F. Supp. 2d at 19.

[22] Economists will recognize this approach as involving "conjectural variations," as developed by A.L. Bowley in The Mathematical Groundwork of Economics (Oxford Univ. Press 1924) and extended by Timothy Bresnahan, *Duopoly Models with Consistent Conjectures*, 71 Am. Econ. Rev. 934 (1982), and Martin K. Perry, *Oligopoly and Consistent Conjectural Variations*, 13 Bell J. Econ. 197 (1982). The theory of conjectural variations has been criticized in economic theory because it does not lay out from first principles a game-theoretic model of oligopoly. This is something of a bum rap. Economic theory does not suggest that oligopolies will tend to be characterized by static Nash competition; it is just that game theorists understand that case best. But the fact is that oligopolists often do respond to rivals' price changes, so a theory that recognizes that fact has some considerable advantage over a more elegant theory that denies it. Moreover, though not easy, it is perfectly possible to analyze price responses in game-theoretically primitive form. *See, e.g.*, Eric Maskin & Jean Tirole, *A Theory of Dynamic Oligopoly II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles*, 56 Econometrica 571 (1988).

[23] The importance of the residual demand curve for assessing market power is not new in antitrust. *See, e.g.*, Jonathan B. Baker & Timothy F. Bresnahan, *Estimating the Residual Demand Curve Facing a Single Firm*, 6 Int'l J. Indus. Org. 283 (1988).

[24] The Appendix explains the technical assumptions underlying Proposition 2. We assume that the products are "strategic complements," so when one firm raises its price it expects the others to raise their prices (or leave them unchanged). We also assume that the prices set by firms in the candidate market are not materially influenced by any responses they anticipate from firms outside the candidate market. Our results would need to be modified if this condition is not met.

In the usual case where firms expect accommodating responses, meaning that rivals at least partly match price changes, less of each firm's lost sales will remain within the candidate market than would do so if those rivals did not respond. Thus, $A^* < A$, so using A and Proposition 1 could incorrectly suggest that a group of products is a market. Intuitively, if premerger firms face substantial accommodating responses, then they are not competing very strongly, so there is less of a change from that situation to that of control by the hypothetical monopolist, who therefore might find a SSNIP unprofitable.[25] Proposition 2 offers a conservative check on that possibility, by giving a sufficient (but not necessary) condition using the smaller value A^*.

The Residual Aggregate Diversion Ratio, A^*, may be estimated from industry experience of responses to price changes. Econometrically, it would be important to ensure that those price changes are indeed initiated by the one firm, perhaps following changes in its firm-specific costs, or changes in managerial assessment of market conditions, as opposed to general price shifts prompted by industry-wide cost shocks or by shifts in demand. Evidence on A^* could also come from documents of the merging parties, especially those analyzing the profitability of a price change by estimating the extent to which sales will *in fact* (in the premerger oligopoly) be lost to (or taken from) various rival products. For example, a merging party might have documents predicting rival pricing responses. One can estimate A^* by combining these expected pricing responses with survey data indicating customer switching patterns in response to price changes. Furthermore, evidence regarding A will bear on A^*, either through the inequality $A^* \leq A$ or to the extent that one might be able to trace through the differences between the two.[26]

Estimation of A^* will often be more reliable than estimation of L, because A^* is directly relevant to real-world premerger pricing decisions faced by the firms: "If we raise our price, and our rivals respond as we expect, how many customers will we lose and what alternatives will they pick?" In contrast, L inherently involves a hypothetical that is typically a significant, artificial departure from the real, premerger world: "If we and a collection of our rivals raise our prices in unison, how many customers will we lose?" This question is unlikely to be contemplated by suppliers in the normal course of business (unless they are engaged in price fixing or the industry dynamics involve very strong price-matching responses).

3. Demand More Sensitive to Price Increases than Price Decreases

Scheffman and Simons criticize the Katz-Shapiro/O'Brien-Wickelgren test in Proposition 1 because it assumes that customers are equally sensitive to price increases and price decreases: "But the price elasticity might be significantly different for price increases than for price decreases."[27] We now show how Propositions 1 and 2 can be modified to address this possibility.

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[25] The Merger Guidelines recognize that premerger coordination may not persist, so the "but-for merger" price from which the SSNIP should be measured may be lower than the premerger price. Section 1.1 in the Merger Guidelines states that "the Agency will use prevailing prices of the products of the merging firms and possible substitutes for such products, unless premerger circumstances are strongly suggestive of coordinated interaction, in which case the Agency will use a price more reflective of the competitive price." With strongly accommodating premerger behavior, it may not be appropriate under the Merger Guidelines to measure the SSNIP starting from premerger prices, and the test in Proposition 2 would need to be modified. Our approach offers a possible way to do so—using the no-reactions Aggregate Diversion Ratio—while as far as we are aware other quantitative techniques do not. If one believes that premerger pricing reflects (perhaps tacit) coordination that ought not to be taken as the but-for world, market definition might begin from a lower price, just as it ought in circumstances threatened by the *Cellophane* Fallacy.

[26] The relationship between A and A^* depends upon the degree to which rivals match a single firm's pricing changes and the impact of those reactions on the firm's sales (as measured by diversion ratios). The proof of Proposition 2 in the Appendix derives a formula for the relationship between A and A^*.

[27] Scheffman & Simons, *supra* note 16, at 6.

Suppose that a single product loses sales in response to a price increase, accounting for responses by other firms in the candidate market, at a rate $1 + k$ times the rate at which it gains sales in response to a tiny price decrease.[28] With this definition, which is simply an arithmetical way to keep track of such possible asymmetry on the residual demand curve, Proposition 2 becomes:

> *Proposition 3:* If each firm owns a single product and prices to maximize its profits accounting for price responses by other firms in the candidate market, a symmetric group of products forms a market under break-even analysis if and only if
>
> $$A^* \geq \frac{\frac{s}{m+s} + k}{1+k}$$
>
> which is equivalent to
>
> $$k \leq k_{max} = \frac{A^* - \frac{s}{m+s}}{1 - A^*}.$$

Begin with a default assumption that demand is equally sensitive to price increases and price decreases . . .

Following Katz and Shapiro,[29] it seems reasonable to require a party arguing for $k > 0$ based on asymmetric customer responses to present evidence in support of this claim.

Another possibility is that customers are equally sensitive to price increases and decreases but *rivals* have asymmetric responses. Suppose, in particular, that rivals more fully match price cuts than they do price increases: this is the "kinked demand curve" theory of oligopoly.[30] The Appendix shows how such behavior can imply $k > 0$ and provides a method of calculating k.[31]

One procedure for incorporating these issues would run as follows. Begin with a default assumption that demand is equally sensitive to price increases and price decreases: $k = 0$.[32] Under this default assumption, Proposition 2 tells us that the symmetric group of products forms a market if and only if $A^* \geq s/(m + s)$. If reliable evidence on A^* implies that this condition holds, there would be a presumption that these products form a market, but this presumption could be rebutted.

One way to rebut the presumption would be to produce other evidence on Actual Loss. If that other evidence suggests that $L > s/(m + s)$, it would conflict with the evidence suggesting that $A^* > s/(m + s)$. In Section 5 we offer some comments on how a court might weigh such conflicting evidence, but our principal point is that it should do so, and should not ignore the evidence on Actual Loss that is based on estimating A^*.

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[28] The Appendix gives a technical definition of k. If the demand curve is not literally "kinked," but only curved, then it is not necessary to specify that the tiny price change is a decrease.

[29] *See* Katz & Shapiro, *supra* note 12.

[30] *See* Paul M. Sweezy, *Demand Under Conditions of Oligopoly*, 47 J. Pol. Econ. 568 (1939). Sweezy refers to the "imagined demand curve," which we now know as the "residual demand curve." *See also* R.L. Hall & C.J. Hitch, *Price Theory and Business Behaviour*, *in* 2 Oxford Econ. Papers 12 (1939).

[31] If a candidate market includes all of the rival products whose asymmetric responses kink the residual demand curve for any one product, then as Katz and Shapiro observe, there is no kink in the demand curve facing the hypothetical monopolist. *See* Katz & Shapiro, *Further Thoughts on Critical Loss*, *supra* note 16.

[32] Alternatively, one might begin with a different default assumption or presumption about k, based on empirical evidence not specific to the case at hand. Although we would not want to over-weight evidence consisting of economic theorists' and econometricians' conventional choices of functional forms for study, we note that while linear demand has $k = 0$, constant-elasticity and several other widely used "standard" demand systems have $k < 0$.

Another way to rebut the presumption would be through evidence that demand is sufficiently more sensitive to price increases than to price decreases. The necessary level of asymmetry k_{max} is given by Proposition 3. One might use a sliding scale of rebuttable presumption, with more convincing evidence required for larger k_{max}. As discussed in our companion paper, evidence on the rate at which changes in costs are passed through to changes in price is relevant to k.[33]

This approach treats evidence concerning demand response (such as company documents, customer interviews, marketing surveys, or econometric estimates of demand) quite differently than does Critical Loss analysis as it is often currently performed. In many cases, such as *Whole Foods*, an expert for the merging parties estimates Actual Loss using evidence that says little or nothing about *asymmetric* demand response, neither explicitly estimating k nor calculating k_{max}, but that tends to suggest that the demand facing the hypothetical monopolist is sensitive to price.[34] Combining such evidence with a calculation that, with fairly high margins, Critical Loss is not very large, some experts and courts have inferred that a group of products is too narrow to be a market. But this inference ignores premerger pricing information: a high demand elasticity would also suggest that firms were pricing irrationally high prior to the merger. Thus, it puts little or no weight on firms' own premerger pricing decisions, or is very willing to believe a high value of k, or both. The approach discussed here, in contrast, discounts evidence that would suggest high demand elasticity if it conflicts with the evidence from firms' premerger pricing decisions, unless that conflict is resolved by evidence on demand response or industry dynamics.

4. Complexities in Real-World Profits

Real-world firms often do not maximize the direct, readily quantifiable profits from any one product considered in isolation. When a firm sells competing or complementary products, it will naturally account for spillover effects on profits when setting the price of any one product. Plus, dynamic and intangible considerations, such as customer loyalty, reputation, network effects, and learning curves, commonly arise.

When such factors are significant, it matters whether or not one views the hypothetical monopolist as inheriting premerger firms' concern for those factors. If so—an assumption that might place more strain on our assumption of symmetry[35]—then we can extend the analysis above relatively simply. If not, then there are reasons to be pessimistic about the likely contribution of market definition to a reasonable presumption or screen concerning a merger's competitive effects.

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[33] Farrell & Shapiro, *supra* note 8.

[34] Criticizing the economic expert for the merging parties, the economic expert for the FTC stated: "Economic theory makes no prediction that consumers will respond more to price increases than to price decreases, and Dr. Scheffman provides zero evidence that such asymmetric responses would be expected in this case, or any other one. All of the qualitative evidence he relies on is equally supportive of large responses to price increases and decreases, both for existing firms and for a hypothetical PNOS monopolist." *See* Rebuttal Expert Report of Kevin M. Murphy ¶ 14, FTC v. Whole Foods Mkt., Inc., 502 F. Supp. 2d 1 (D.D.C. 2007) (Civ. No. 07-cv-01021-PLF), *available at* http://www.ftc.gov/os/caselist/0710114/070823rebutmurphy.pdf. A different way in which direct estimates of (or claims about the magnitude of) Actual Loss sometimes fail to confront contrary evidence from revealed preference is if the direct estimate merely recites a number of ways by which demand elasticity *could* arise—ways in which customers *could* substitute—but does not reconcile this estimate with the evidence from premerger margins that those channels of substitution do not *in fact* cause high premerger elasticity.

[35] Of course, these complexities may differ for the different firms in the market. As noted above, in practice, Critical Loss Analysis is performed by considering a uniform SSNIP applied to symmetric products. We retain the assumption of symmetry in this section and refer readers to Moresi et al. and to our companion paper, *Cannibalization, Pass-Through, and Market Definition*, for analysis of asymmetric situations. *See* Moresi, Salop & Woodbury, *supra* note 7; Farrell & Shapiro, *supra* note 8.

Suppose that the various factors that enter into the premerger firms' profit functions are also included in the hypothetical monopolist's profit function. In the symmetric case, this seems the most useful and appropriate way to interpret the hypothetical monopolist test.[36] After all, the point of the test is to identify the collection of firms that would find it profitable to impose a SSNIP, so that changes in concentration among that group of firms tell one something about the likely competitive effects of the merger. Concern for customer reputation or for follow-on sales of a tangible complement applies before the proposed merger and will apply afterwards. Therefore, it seems natural to include it in any analysis intended to be informative about competitive effects.[37]

The calculations above did not take such factors into account, either in the simplified profit function used for the hypothetical monopolist or for the profit functions of the premerger firms. We now show how to use premerger pricing information in markets where these complexities are important.

To illustrate, suppose that the premerger firms all sell a complementary product as well as the primary product being studied. Suppose that each firm expects some profitable sales of the complement to follow each sale of the primary product. For example, in a candidate market for enterprise computers, each manufacturer may anticipate profitable follow-on sales of information technology management services to the buyers of its computers. Then a premerger firm sets the price of its primary product to maximize not simply the direct profit from sales of the primary product, $(p - c)x$, but rather $(p - c)x + Bx$, where B represents the anticipated follow-on benefits (profits) from selling one more unit of the primary product.[38] This formulation is general enough to handle a wide range of complexities, including network effects, learning-by-doing, and reputation effects, as well as multi-product firms. An outside observer may or may not be able to estimate accurately the benefit B in a particular case, but even when it is difficult to do so, it may at least be feasible to determine whether B is positive or negative. We believe that most of the intangibles that businesses typically focus on involve a positive value of B, so we assume here that $B \geq 0$. That is, if anything, businesses typically like to make sales even past the point where a simple quantitative profit analysis would recommend.

Introducing additional per-unit benefits of B is economically equivalent to reducing the net cost of selling a unit from c to $c - B$. The true economic margin is thus $m^* = (p - c + B)/p$ rather than $m = (p - c)/p$, the accounting margin. We write $m^* = m + b$ where $b = B/p$. Replacing the accounting margin m with the economic margin m^*, all of the analysis leading up to Propositions 1, 2, and 3 can be repeated, but now the presence of these follow-on (perhaps intangible) benefits does not undermine the Lerner equation, which takes them into account. Thus Propositions

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[36] The Merger Guidelines do not explicitly state what factors should be included in the profit function of the hypothetical monopolist, and we are not aware of this issue being addressed in other published work or agency guidance.

[37] Scheffman and Simons emphasize that the Critical Loss is "just arithmetic." Scheffman & Simons, *supra* note 16, at 2, 3, 4. That is only true if one uses the highly simplified profit function for the hypothetical monopolist. While using that profit function is reasonable if the excluded factors are small, when they are large it introduces a significant change in incentives having nothing to do with the hypothesized end to competition among the products in the candidate market. It thus muddies the waters of market definition. We show below how using the simplified profit function can lead to absurd results. *See also* David S. Evans & Michael D. Noel, *Analyzing Market Definition and Power in Multi-Sided Platform Markets* (Working Paper Oct. 21, 2005), *available at* http://ssrn.com/abstract=835504 (arguing that standard techniques applied to one side of a platform market in isolation are often badly misleading; one can view this as a form of omitting consideration of an important complement).

[38] We continue to assume that the firm maximizes its overall profits. Scheffman and Simons observe that firms do not always exactly maximize profits. *See* Scheffman & Simons, *supra* note 16. This is surely true, but the critical-loss methodology, and indeed almost all modern antitrust economics, is based on evaluating firms' profit incentives (and the Merger Guidelines follow this approach regarding the hypothetical monopolist). Because merger analysis focuses on the *change* in market power, a disciplined approach should presumably take the same view of the profit maximization hypothesis (or approximation) before and after a merger.

1, 2, and 3 remain valid, replacing m with $m^* = m + b$. For example, the condition in Proposition 2 becomes

$$A^* \geq \frac{s}{m^* + s} = \frac{s}{m + b + s}.$$

If one can reliably quantify b then one can use these formulae directly to diagnose antitrust markets in this more complex environment. In other cases, it will not be possible to quantify b and thus calculate m^*. However, if $B > 0$ then $m^* > m$, so if the conditions provided in Propositions 1, 2, and 3 hold using the accounting margin m, then they hold using m^*, so one can all the more strongly conclude that the candidate group of products form a market.

> *Proposition 4:* If each firm receives some additional benefit from selling its primary product, and if these benefits are not included in the accounting margin m, then if the conditions in Propositions 1, 2, and 3 hold using that accounting margin, the candidate group of products must form a market under break-even analysis, so long as these benefits also are included in the profits of the hypothetical monopolist.

We now address the possibility that $B < 0$, i.e., that each firm incurs some cost, not measured in the accounting margin, when it sells one more unit of the primary product. This possibility arises if the primary product cannibalizes sales from substitute products owned by each firm that are not in the candidate market.[39] In this situation, the tests in Propositions 1, 2, and 3 can incorrectly report that a candidate group of products is a market: the true economic margin is lower than the accounting margin, so the elasticity of demand for each product is higher than would be inferred from the Lerner Equation using the accounting margin, leading to a larger Actual Loss. One way to analyze that situation is to perform Critical Loss Analysis on a broader group of products including those substitute products.[40] Alternatively, if the cannibalization effect can be measured, one can use the (lower) true economic margin instead of the accounting margin in Propositions 1, 2, and 3.

While we think it makes sense to include B in principle in the hypothetical monopolist's profit function in the symmetric case, one might read the Merger Guidelines not to do so. Surprisingly, the issue does not seem to have attracted explicit discussion. We do not take a strong general position here, but simply observe that omitting a substantial B can be expected to lead to market definitions that may be uninformative or misleading.[41]

The fundamental idea of market definition for merger analysis in unilateral effects cases is to provide a simple preliminary gauge of the extent to which a merger's removal of competition between the merging firms' products will threaten competition. To do so, it asks how important that competition is in restraining prices, premerger. It seems strikingly uninformative to observe, for instance, that if, instead of removing that competition, one severed each product's link to an important complement that is present premerger and will be present postmerger, then prices would rise substantially. But this is all the hypothetical monopolist test would be telling us if we stripped the hypothetical monopolist of premerger firms' concerns for their complements: in fact, if $b > s$, one would find single-product markets.[42]

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[39] Moresi et al. focus attention on this particular case. *See* Moresi, Salop & Woodbury, *supra* note 7.

[40] Strictly speaking, doing this might not adhere to the algorithm in the Merger Guidelines for adding products in the order of "next-closest substitutes," and "generally" stopping with the smallest group of products that forms a market. In practice, this algorithm is rarely followed slavishly, in part because the information necessary to do so is typically unavailable (and the "narrowest-market principle" can be misleading).

[41] *See also* Evans & Noel, *supra* note 37.

[42] In general, with $B > 0$, the Appendix shows that the hypothetical monopolist test systematically leads to narrower markets if the hypothetical monopolist's profits do not include B than if they do.

5. Conflicting Evidence

Because market definition is often central in antitrust litigation, courts and agencies are often confronted with conflicting Critical Loss analyses. We offer some suggestions for using revealed preference analysis to help confront such conflicting economic evidence.

Suppose that a SSNIP of 5 percent will be used and that there is no dispute that the accounting margin is 45 percent.[43] If there are no significant intangible benefits, these values for m and s imply that the Critical Loss is 10 percent. Suppose, however, that the opposing experts disagree over the Actual Loss to be expected if a hypothetical monopolist over the government's proposed market were to impose a SSNIP.

Professor A, the government's economic expert, performs a study finding that the Actual Loss would be only 8.3 percent. Because this is less than the Critical Loss of 10 percent, she finds that the government's proposed market is indeed a market.

Professor B, the expert for the merging parties, performs a different study concluding that the Actual Loss would be L = 15%. Because this exceeds the Critical Loss of 10 percent, he testifies that the government's proposed market is not a market.

Can we help the judge assess the reliability of the conflicting estimates?

If neither study used revealed-preference methods, one can ask what each would imply for the Residual Aggregate Diversion Ratio, A^*. To do so, we ask what loss of sales an individual product would experience if it unilaterally raised its price by the 5 percent SSNIP. Given the margin of 45 percent, the Lerner equation suggests an elasticity of residual demand of 1/.45, or 2.2. With linear demand, the loss of sales from a 5 percent price increase would be 2.2 times 5 percent, or 11 percent. This is less than the 15% Actual Loss estimated by Professor B, the defense expert. While the Actual Loss for the single producer is greater than the Critical Loss, how can the Actual Loss for the hypothetical monopolist be *more* than the loss for a single product?

As Section 3 discussed, Professor B could defend his L = 15% estimate of Actual Loss by arguing that demand is more sensitive to price increases than price decreases. With such demand asymmetry, the Lerner equation for premerger firms gives an estimate of Actual Loss for the hypothetical monopolist of $L = (1 + k)(1 - A^*)(s/m)$. Since $s/m = 0.11$, Professor B's estimate that $L = 0.15$ implies that

$$(1 + k)(1 - A^*) = \frac{0.15}{0.11} = 1.36, \text{ or } k = \frac{1.36}{1 - A^*} - 1,$$

so $k > .36$ (so long as A^* is positive). While k might be this large, as discussed in Section 3, we would recommend further probing of Professor B's Actual Loss estimate, since it implies such a sharply curved (or kinked) residual demand curve.

Defense expert Professor B might alternatively point out that many factors other than accounting profits enter into firms' premerger pricing decisions. Our analysis in Section 4 implies that this

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[43] Since m is important in Critical Loss Analysis, one might expect it to be controversial, as it typically is in predatory pricing cases. If Actual Loss is estimated separately, then estimates of m are used only in calculating Critical Loss, where a higher estimate of m lowers estimated Critical Loss and thus tends to lead to broader markets. Typically one would then expect to see the government arguing for lower m and the merging firms for higher m, although it is not always the case that broader markets assuage competitive concerns. If, on the other hand, Actual Loss is estimated from revealed preference, then a higher estimated m reduces the estimates of both Critical Loss and Actual Loss; as it turns out (see Propositions 1 to 3) a higher estimate of m tends on balance to lead to narrower markets. Strictly, the relevant cost concept is average incremental cost over the demand increment concerned. Average incremental cost is, by definition, equal to the average value of marginal cost. In practice it is often approximated by accounting-based measures of average variable cost (see, e.g., Harris & Veljanovski, *supra* note 4, at 214), although this approximation is not always reliable. One can view some of the analysis in this section as also offering insights into the effects of simply over- or under-estimating gross margins.

is not a good answer if $b > 0$, as we expect will normally be the case. For then the true economic margin exceeds the accounting margin of 45 percent, so the Lerner equation indicates that the elasticity of demand facing any given product is *lower* than the previous estimate of 2.2. For example, if selling one more unit generates intangible benefits valued at 5 percent of the price, so $b = 5\%$, then the economic margin is $m^* = m + b = 45\% + 5\%$, so the Lerner equation implies that the elasticity of demand facing any one product is 2.0. Therefore, the loss of sales for a single product following a unilateral SSNIP will be 10 percent, which casts even more doubt on Professor B's Actual Loss estimate of $L = 15\%$. Thus a challenge to the use of the Lerner Equation here must involve either a claim that premerger firms were pricing *above* their profit-maximizing level (and our use of residual demand implies that this would not be simply a result of oligopoly interdependence), or face important intangibles discouraging additional sales, or else a radical challenge to the use of the profit maximization methodology.

With the numbers given above, government expert Professor A's estimate that $L = 8.3\%$ is easily calculated to be consistent with an Aggregate Diversion Ratio of 25 percent if k is zero. Specifically, recall that the Lerner equation applied to a 45 percent gross margin implies a product-level residual demand elasticity of 2.2. If $A^* = 0.25$ then the predicted demand elasticity for the hypothetical monopolist is $(1 - 0.25) \times 2.2 = 1.65$, so that a 5 percent SSNIP would lead to an Actual Loss of $1.65 \times 5\% = 8.3\%$. The court could ask whether this 25 percent estimate for A^* is consistent with the evidence.[44]

Another possibility is that one of the experts relied on the revealed-preference method. Suppose for instance that the government's expert, Professor A, put less weight on marketing and econometric studies of demand but conducted a study of demand substitution patterns in response to price changes initiated by one product. Her study yielded an estimate $A^* = 0.25$, and she testified that this was her primary basis for her $L = 8.3\%$ estimate. In this case, obviously, consistency with A^* is not an issue. Now the two experts have arrived at their conflicting estimates of L via more fundamentally differing paths. Professor A estimated A^*, inferred L, and found that the inferred estimate of L was less than the Critical Loss. Professor B estimated L to be greater than the Critical Loss. They embody the two bullet points in Section 1.B.

In this situation, we hope the court would recognize and confront the tension between the two approaches. Naturally, the court will want to assess the quality of the evidence underlying the estimate of $A^* = 25\%$ by the government's expert and the estimate of the Actual Loss of $L = 15\%$ by the expert for the merging parties.

The court may be tempted to give greater weight to the separate "direct" estimate of L than to the indirect estimate of Actual Loss based on an estimate of A^* combined with use of the Lerner Equation. However, we believe such an approach would not be justified, for three reasons. First, the version of the Lerner Equation developed in this paper is very general and flexible, allowing for oligopolistic interactions, for curvature of demand, and for many other factors to enter into firms' pricing decisions. Rejecting this version of the Lerner Equation is tantamount to rejecting the whole profit-maximization methodology that underlies the Merger Guidelines and indeed much if not all of antitrust economics as currently practiced. Second, as noted above, it may well be possible to obtain a more reliable estimate of A^* than of L based on documentary evidence, because A^* relates more closely to questions that the suppliers actually face in the real, premerger world. Third, since Critical Loss Analysis involves comparing two estimated quantities (the Critical and

[44] The court also can ask what value of k would bring the estimate of A^* in line with the other evidence.

Actual Losses), its accuracy depends in potentially complex ways on the accuracy of the two estimates. In particular, if a method errs in the same direction in its estimation of each quantity, it may be more robust and accurate than another method whose errors are less correlated. Further work on this issue could prove helpful.

6. Break-Even and Profit-Maximizing Tests

As we noted in the previous paragraph, the assumption of profit maximization permeates much, if not all, of antitrust economics as currently practiced. The break-even version of Critical Loss Analysis we have explored so far does not analyze what a hypothetical monopolist would find *most* profitable, but only asks which of two pricing patterns it would find *more* profitable. In doing so, it departs not only from usual economic and antitrust methodology, but also from the Merger Guidelines themselves. Such a departure is somewhat built into an approach of separate estimation of Critical and Actual Losses, but is avoidable if one is willing to take as a working approximation that demand is roughly linear or to use another functional form for demand. Thus, we can modify our Propositions 1 through 4 to work with the profit-maximizing version of the SSNIP test called for in the Merger Guidelines.

A. Relationship Between the Two Tests. The Appendix provides mild conditions such that if the profit-maximizing response to hypothetical monopolization is to impose at least a SSNIP, then a SSNIP is more profitable than the status quo.[45] Intuitively, if a SSNIP is a partial move toward profit maximization (from the status quo) then it will likely yield some increase in profit. However, even when these conditions are met, the converse need not hold, and no reasonable conditions suggest themselves under which one could make that inference. Thus, for a given size of SSNIP, the break-even test is easier to satisfy, and will tend to suggest narrower markets.

Further work is needed to learn more about which of the two tests is more robust, i.e., less sensitive to errors in the measurement of the Actual Loss, the Residual Aggregate Diversion Ratio, the curvature of the residual demand curve, and the premerger margins.

B. Linear Demand: Modifying Propositions 1 and 2. Recall that Propositions 1 and 2 assumed linear (residual) demand. The Appendix shows that for a firm—including a hypothetical monopolist—with constant marginal cost and linear (residual) demand, the profit-maximizing price change from any status quo is just half the break-even change. Thus, as Katz and Shapiro noted,[46] it is profit-maximizing to impose (at least) a SSNIP s if and only if a price increase of $2s$ is more profitable than the status quo. As a result, we can readily modify Propositions 1 and 2 as follows:

> *Proposition 1A (Katz-Shapiro-O'Brien-Wickelgren):* If each firm owns a single product and prices to maximize its profits taking as given all other prices, and if demand for each product is linear in price for small price changes from the premerger price, then a symmetric group of products forms a market under profit-maximizing analysis if and only if
>
> $$A > \frac{2s}{m + 2s}.$$

[45] These conditions will often be met, but not always. Discussing the *Sungard* case, Katz notes that they would not be met if a substantial number of customers would switch in response to a 5 percent SSNIP, making it unprofitable, but few additional customers would switch in response to a considerably larger price increase, making the latter profit-maximizing. *See* Katz, *supra* note 4. He also suggests that even if *in fact* a 5 percent SSNIP would be profitable, it might be easier to *prove* that a larger price increase would be profitable.

[46] *See* Katz & Shapiro, *supra* note 12.

Proposition 2A: If each firm owns a single product and prices to maximize its profits accounting for price responses by other firms in the candidate market, and if demand for each product is linear in price for small price changes starting from the premerger price, then a symmetric group of products forms a market under profit-maximizing analysis if

$$A^* \geq \frac{2s}{m+2s},$$

but may do so even if that condition fails to hold.

The logic underlying Proposition 4 carries over to the profit-maximizing version of the SSNIP test, so we have

Proposition 4A: If each firm receives some additional benefit from selling its primary product, and if these benefits are not included in the accounting margin *m*, then if the conditions in Propositions 1A and 2A hold using that accounting margin, the candidate group of products must form a market under profit-maximizing analysis, so long as these benefits also are included in the profits of the hypothetical monopolist.

Our overall message is that the complexities sometimes thought to undermine inferences based on premerger margins need not do so.

C. Demand More Sensitive to Price Increases: Modifying Proposition 3. If demand is more sensitive to price increases than price decreases, the profit-maximizing analysis is significantly more complex than the break-even analysis. The profit-maximizing price increase depends upon the precise shape of the demand curve at prices above the premerger price. Our companion paper explores how the rate at which cost changes are passed through into price changes features in this analysis and develops tests for market definition with non-linear demand using the profit-maximizing SSNIP test in the Merger Guidelines.[47]

Conclusion

We have developed new and improved tools for using premerger pricing information in Critical Loss Analysis to help define markets under the Merger Guidelines. Our overall message is that the complexities sometimes thought to undermine inferences based on premerger margins need not do so. We have explored three such complexities: various modes of oligopolistic interaction; greater sensitivity of demand to price increases vs. price decreases; and various complementarities and business intangibles that enter into firms' profits. We have provided two versions of our results: the analysis that applies to the breakeven version of Critical Loss Analysis that is used regularly in merger litigation, and one that applies to the profit-maximizing version of the SSNIP test actually called for in the Merger Guidelines.

In Section 2, we extended the analysis to cases in which it is important to account for the possibility that oligopolists respond to one another's price initiatives. Katz and Shapiro[48] discussed this but we take the analysis further here. Proposition 2 shows that the natural generalization of Propositions 1 to dynamic oligopolistic conduct continues to provide sufficient conditions for a group of products to form a market.

In Section 3, we extended Proposition 2 to allow for the possibility that residual demand is more sensitive to price increases than to price decreases. We calculated how much asymmetry in the price sensitivity of residual demand must be present for a group of products *not* to form a market. This result, Proposition 3, relies only on Critical Loss arithmetic and the standard assumption

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[47] *See* Farrell & Shapiro, *supra* note 8.

[48] Katz & Shapiro, *supra* note 12.

that the premerger price of each product in the candidate market was set to maximize the profits from that product. So far as we are aware, no such general condition has previously been available. We suggest that the agencies and the courts should require convincing evidence before accepting claims that residual demand is much more sensitive to price increases than to price decreases. If an expert estimates an Actual Loss for a hypothetical monopolist implying that premerger firms were failing to maximize profits premerger, that result should be explicitly acknowledged and some skepticism applied. Unfortunately, this does not appear to have happened in *Whole Foods*.

If an expert estimates an Actual Loss for a hypothetical monopolist implying that premerger firms were failing to maximize profits premerger, that result should be explicitly acknowledged and some skepticism applied. Unfortunately, this does not appear to have happened in Whole Foods.

In Section 4, we asked how different approaches fare when business considerations omitted from the simplified profit function substantially affect premerger (and likely postmerger) pricing. We showed that when intangibles like customer goodwill favor low prices, as we would typically expect, the conditions in Propositions 1, 2, and 3, calculated using accounting margins, remain sufficient for a candidate group of products to form a market so long as the profits of the hypothetical monopolist also include these same intangible factors. This is a major generalization of previous results. Section 6 extends the results to a profit-maximizing SSNIP.

Larger premerger margins imply a lower Critical Loss but also, by revealed preference, strongly suggest a lower value of Actual Loss. Using the revealed-preference approach, larger premerger margins on balance suggest narrower markets, in contrast to what is suggested if one estimates Actual Loss separately. This lesson can be seen in Propositions 1, 2, and 3, which show that a smaller degree of competition among the products in a candidate market (as measured by the Aggregate Diversion Ratio) is sufficient for these products to form a market if the premerger margin is larger. Intuitively, larger premerger margins magnify the cannibalization effect, making it more profitable for the hypothetical monopolist to impose a SSNIP. Another lesson is that markets are narrower, the smaller is the SSNIP used. Section 5 illustrates how the analysis here can be used to test the market definition claims of opposing experts.

Some of our analysis is fairly simple, some not. But where it gets complicated, it does so because it is dealing with complicated reality. One can undertake Critical Loss Analysis in a seemingly simpler manner, but only by ignoring key revealed-preference information and stripping the hypothetical monopolist of key business concerns shared by pre- and post-merger actual firms, thus depleting the market definition exercise of both reliability and relevance.

Even with the new and improved tests developed here and in our companion paper,[49] the whole market definition exercise has some serious drawbacks. We argue in ongoing work[50] that an alternative approach can be used instead of the market definition and market concentration methodology in cases where the primary concern is unilateral anticompetitive effects. We argue that this alternative approach offers a simple and intuitive formulation of unilateral effects that is far more transparent and robust than traditional merger simulation and encourages the integration of marginal-cost efficiencies at an early stage in the analysis. ●

[49] *See id.*

[50] *See supra* note 5.

Appendix

Critical Loss Calculation

If the hypothetical monopolist would lose sales of *Z* units, then the SSNIP increases the (simplified) profits of the hypothetical monopolist if and only if $(p+sp-c)(X-Z)>(p-c)X$, i.e., if the profits earned on the remaining sales at the higher price exceed those earned on the initial sales at the lower price. This expression can be rewritten as $Z/X<s/(m+s)$.

Proof of Proposition 1

Let the (point) elasticity of demand facing a single product be given by ε. By the Lerner Equation, $\varepsilon=1/m$. If a single firm raises the price of its product by a SSNIP, and if the demand for its product is linear over this range in prices, the percentage decrease in its sales will be equal to the percentage increase in price, *s*, times the elasticity, ε. Therefore, its percentage loss of sales will be εs, which equals s/m.

Now consider the hypothetical monopolist that imposes a uniform SSNIP on all of the products in the candidate market. The hypothetical monopolist will recapture a fraction *A* of the sales lost by any one product when its price is raised, since those lost sales will be diverted to products owned by the hypothetical monopolist. Therefore, Actual Loss for the hypothetical monopolist, measured as a percentage of initial sales, will be $(1-A)\varepsilon s$ which equals $(1-A)s/m$. The products form a market if and only if this expression is less than the Critical Loss of $s/(m+s)$. This simplifies to $A\geq s/(m+s)$, proving Proposition 1.

This proof relies on the assumption of linear demand. We used linear demand to conclude that the single firm's loss in sales will be εs, an expression that is linear in the price increase. If demand is convex, the loss of sales will be smaller and the condition in Proposition 1 will be sufficient but not necessary for the products to form a market. Concave demand is studied in Proposition 2.

The proof also relied on the assumption that the Aggregate Diversion Ratio is constant over the relevant range of prices (between premerger prices and the prices after the SSNIP). We used this assumption when we stated that the Actual Loss for the hypothetical monopolist imposing a uniform SSNIP will be $(1-A)\varepsilon s$. One can think of the hypothetical monopolist as imposing the SSNIP on one product after another *seriatim*. The resulting recapture rate will be *A* if the Aggregate Diversion Ratio is the same for each individual price increase.

Proof of Proposition 2

We show here that the Actual Loss as a fraction of initial sales is at least $(1-A^*)/m$.

We define δ as the diversion ratio between any pair of products, i.e., the fraction of sales lost by one product when its price alone rises that are captured by the other product. We assume that all the diversion ratios are constant in the relevant range and that each firm's demand is linear in price over the relevant range. So we can write the unit sales of Product 1 as $x_1=\bar{x}_1-(p_1-\bar{p}_1)+\delta\sum_{j\neq 1}(p_j-\bar{p}_j)$. The Aggregate Diversion Ratio is $A=\delta(K-1)$ where there are *K* products. If the prices of all products other than Product 1 are the same, we can write $x_1=\bar{x}_1-(p_1-\bar{p}_1)+A(p_2-\bar{p}_2)$. We also can write the change in output at the other firms when Product 1's price alone changes as $x_2=\bar{x}_2-(p_2-\bar{p}_2)+\delta(p_1-\bar{p}_1)+\delta\sum_{j\neq 1,2}(p_j-\bar{p}_j)$. With symmetry among all of the other firms, this becomes $x_2=\bar{x}_2+(p_2-\bar{p}_2)[-1+A-\delta]+\delta(p_1-\bar{p}_1)$.

Now we are ready to consider an arbitrary exogenous increase in p_1 and see how the outputs of all firms respond. Call $p_1=\bar{p}_1+h$ and $p_2=\bar{p}_2+\lambda h$ where this defines the matching rate $\lambda\leq 1$. So, we get $x_1=\bar{x}_1-h+A\lambda h$. The loss of sales at Product 1 are $\bar{x}_1-x_1=h(1-\lambda A)$, so the loss rate is $g^*=1-\lambda A$. The gain in sales at Product 2 is $x_2-\bar{x}_2=(p_2-\bar{p}_2)[-1+A-\delta]+\delta(p_1-\bar{p}_1)$ which can be written as $x_2-\bar{x}_2=\lambda h[-1+A-\delta]+\delta h$. The total gain in sales by all of the other firms thus equals $X_{-1}-\bar{X}_{-1}=(K-1)h\{\lambda[-1+A-\delta]+\delta\}$.

The Residual Aggregate Diversion Ratio is defined as $A^* \equiv \frac{X_{-1} - \bar{X}_{-1}}{\bar{x}_1 - x_1} = (K-1)\frac{\lambda[-1+A-\delta]+\delta}{(1-\lambda A)}$. Writing this in terms of primitives gives $A^* = (K-1)\frac{(\delta-\lambda)+\lambda\delta(K-2)}{1-\lambda\delta(K-1)}$. Note as a check that if $\lambda = 0$ this does give back $A^* = \delta(K-1) = A$. Note also that with two firms, $K = 2$, we get $A^* = \frac{\delta-\lambda}{1-\lambda\delta}$. We focus on the case where $A^* > 0$ which is equivalent to $\lambda(1-\delta(K-2)) < \delta$.

Next, we measure the Actual Loss for the hypothetical monopolist who raises prices of all products uniformly by an amount *h*. Product 1's lost sales are $\bar{x}_1 - x_1 = h(1-\delta(K-1))$, which equals $h(1-A)$. Total lost sales are thus $Kh(1-\delta(K-1))$. Note that the hypothetical monopolist's proportional losses are the single firm loss rate (unity with our normalizations) times one minus the aggregate diversion ratio.

Our method of using revealed-preference information requires that we compare the Actual Loss by the hypothetical monopolist with the loss that a single firm would incur given equilibrium matching by its rivals. We explore using the following *estimate* of the sales lost on each product by the hypothetical monopolist: $(\bar{x}_1 - x_1)(1-A^*)$. This is the loss of sales at Product 1 if it raises its price and others follow, which is $h(1-\lambda A)$ times $1-A^*$.

Using this estimate, estimated total lost sales are $K(\bar{x}_1 - x_1)(1-A^*)$. Using $\bar{x}_1 - x_1 = h(1-\lambda A)$ from above, this estimate equals $Kh(1-\lambda A)(1-A^*)$. Since $A = \delta(K-1)$, this estimate is $Kh(1-\lambda\delta(K-1))(1-A^*)$.

This will *over-estimate* lost sales if and only if $Kh(1-\lambda\delta(K-1))(1-A^*) > Kh(1-\delta(K-1))$, which becomes $[(1-\lambda\delta(K-1)](1-A^*) > 1-\delta(K-1)$. Expanding the left-hand side, this becomes $1-\lambda\delta(K-1)-[1-\lambda\delta(K-1)]A^* > 1-\delta(K-1)$ or $(K-1)\delta(1-\lambda) > A^*[1-\lambda\delta(K-1)]$. This is the same as $\delta(1-\lambda) > \frac{A^*[1-\lambda\delta(K-1)]}{K-1}$. Substituting for the right-hand side using the definition of A^*, this becomes $\delta(1-\lambda) > (\delta-\lambda)+\lambda\delta(K-2)$. Simplifying, this becomes $1-\delta(K-1) > 0$, i.e., $\delta(K-1) < 1$: the Aggregate Diversion Ratio is less than unity.

Since we will over-estimate the lost sales by the hypothetical monopolist, if we find that the hypothetical monopolist earns greater profits by imposing the price increase than at the status quo, this comparison will certainly hold using the Actual Losses, which are lower than the estimated losses. Summarizing gives Proposition 3, a sufficient (but not necessary) condition for a group of products to form a market using the breakeven Critical Loss test.

Proof of Proposition 3

The asymmetry parameter *k* is defined so that the proportionate losses facing a single product that raises its price by a SSNIP are $(1+k)(s/m)$, where *s* is the chosen SSNIP (thus in general k will depend on the choice of SSNIP) and the (point) elasticity of demand facing the product is (by the Lerner equation) $1/m$. With this definition, the proportional loss of demand for the hypothetical monopolist following a SSNIP is $(1-A^*)(1+k)(s/m)$. Thus the Actual Loss is less than the Critical Loss if and only if $(1-A^*)(1+k)(s/m) \le s/(m+s)$. Solving for *A* gives

$$A^* \ge \frac{\frac{s}{m+s}+k}{1+k}.$$

With $k = 0$ this simplifies to $A^* \ge s/(m+s)$ as in Proposition 2. The larger is *k*, the higher must be the Aggregate Diversion Ratio for the products to form a market.

To illustrate, with $s = .05$ and $m = 0.45$, $s/(m+s) = .05/(.45+.05) = 0.1$, so a group of products forms a market if and only if $A^* \ge (0.1+k)/(1+k)$. As noted above, for $k = 0$, the test is $A^* \ge 0.1$. For $k = 0.1$, i.e., losses from price increases are 110% of corresponding gains from price decreases, the test becomes roughly $A^* \ge 0.18$. For $k = 0.2$, i.e., losses from price increases are 120% of corresponding gains from price decreases, the test becomes $A^* \ge 0.25$.

We can solve the above expression for *k* to calculate the critical asymmetry ratio, $k_{\max}$, i.e., the minimum amount of asymmetry that would be necessary to lead to a different conclusion:

$$k_{\max} = \frac{A^* - \frac{s}{m+s}}{1-A^*},$$

which gives Proposition 3. Again using $s = .05$ and $m = 0.45$, we have $s/(m+s) = 0.1$, so $k_{\max} = (A^* - 0.1)/(1-A^*)$. If $A^* = 0.2$, then $k_{\max} = .125$. If $A^* = 0.4$, then $k_{\max} = .5$.

Kinked Demand Curve Theory of Oligopoly

Suppose that rivals match price decreases at a rate λ_D and price increases at a rate $\lambda_I < \lambda_D$. A firm that decreases price by *h* gains sales of $h(1-\lambda_D A)$. A firm that increases price by *h* loses sales of $h(1-\lambda_I A)$. Therefore, $1+k = \frac{1-\lambda_I A}{1-\lambda_D A}$, so $k = \frac{(\lambda_D - \lambda_I)A}{1-\lambda_D A}$. With $\lambda_D = 1$ and $\lambda_I = 1/2$ this becomes $k = \frac{A}{2(1-A)}$. With $A = 0.2$ this gives $k = 1/8$.

Exclusion of Additional Benefit in the Profits of the Hypothetical Monopolist

If we do not include these benefits in the hypothetical monopolist's profits function, then the hypothetical monopolist has a marginal cost of *c* rather than $c - B$. Higher costs lead to higher prices, so this must lead to narrower markets than would inclusion.

Following the logic of Proposition 1, the point elasticity of demand facing a single product is now given by $\varepsilon = 1/(m+b)$. With linear demand, if a single firm raises the price of its product by a SSNIP, the percentage loss will be εs, which equals $s/(m+b)$. The Actual Loss for the hypothetical monopolist will be $(1-A)\varepsilon s$ which equals $(1-A)s/(m+b)$. Excluding the benefits from the hypothetical monopolist's profit function, the products form a market if and only if this expression is less than the Critical Loss of $s/(m+s)$. This simplifies to $A \geq (s-b)/(m+s)$. With $b > s$ we get single-product markets.

Profit-Maximizing Price Increase vs. Break-Even Price Increase

So long as the profit function is single-peaked in price, the profit-maximizing price increase will be no more than the break-even price increase. With linear demand and constant marginal costs, profits are a quadratic function of price. It is a property of a quadratic profit function that the profit-maximizing price increase is half as large as the break-even price increase.