vGUPPI: SCORING UNILATERAL PRICING INCENTIVES IN VERTICAL MERGERS

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APPENDIX

I. THE TECHNICAL UNDERPINNINGS OF vGUPPI ANALYSIS

The vGUPPIs can be derived from a formal economic model of upstream competition among input suppliers and downstream competition among output manufacturers.

A. BASIC MODEL WITH DIFFERENTIATED INPUTS AND DIFFERENTIATED PRODUCTS

The formal model is a three-stage game involving *N* input suppliers and *M* output manufacturers. In the first stage, the suppliers simultaneously set the prices of their inputs and each supplier can set different prices to different manufacturers.[1] In the second stage, each manufacturer observes *only* the input prices at which it will be able to purchase inputs from the suppliers; it does not observe the input prices that the suppliers are charging the other manufacturers. The manufacturers then simultaneously set their product prices. These prices determine each manufacturer's product demand, that is, the orders that the manufacturer receives from its own customers. In the third stage, each manufacturer chooses how many inputs to purchase from each supplier, given the volume of orders received and the input prices set by the suppliers. Production and delivery then occur, and the game ends.

[1] We assume suppliers set linear prices and manufacturers have no bargaining power over input prices.

We assume that each supplier and manufacturer chooses its price to maximize profits,[2] and that manufacturers also choose inputs to minimize their costs.[3] The equilibrium concept considered in this analysis is *perfect Bayesian equilibrium*.[4] We thus analyze the model using *backward induction*.[5]

1. *Manufacturers' Purchases of Inputs*

Each manufacturer faces a standard cost-minimization problem (in stage 3). Specifically, manufacturer *m* must fulfill a given volume Q_m of orders (that it received in stage 2) and faces given input prices W_m^1, W_m^2, …, W_m^N from supplier 1, supplier 2, …, supplier *N*, respectively (that the suppliers set in stage 1). We assume that the manufacturer must purchase from the *N* suppliers a total volume of inputs equal to $A_m Q_m$, that is, one unit of output of manufacturer *m* requires A_m units of input.[6] We further

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[2] We assume no coordination both before and after the merger.

[3] An alternative model specification would be to assume that different inputs are used to create multiple differentiated products. Input substitution patterns would involve the interaction of customers' preferences and output prices (as opposed simply to manufacturers' technologies and input prices). We expect that this specification would lead to similar qualitative results.

[4] Because manufacturers do not observe the input prices offered to their competitors, each manufacturer must form beliefs about the input prices paid by other manufacturers. In equilibrium, those beliefs must be correct. In general, there are many perfect Bayesian equilibria because there are many different ways that beliefs can be specified following an unexpected (off equilibrium) input price offer. In this paper, we assume that each manufacturer has "passive beliefs" about the input prices offered to its competitors. This means that, when a supplier contemplates raising the price of its input to a given manufacturer, the supplier expects that the manufacturer will not change its beliefs with respect to the input prices that the supplier is offering to its competitors (and hence that the manufacturer also will not change its beliefs with respect to the output prices that its competitors will set in the downstream market). This assumption of passive beliefs affects the manufacturer's response to an input price increase, and hence the equilibrium level of input prices. *See* Patrick Rey & Jean Tirole, *A Primer on Foreclosure*, *in* 3 HANDBOOK OF INDUSTRIAL ORGANIZATION 2145, § 2.1.2 (Mark Armstrong & Robert H. Porter eds., 2007).

[5] For a formal treatment of the concepts of perfect Bayesian equilibrium and backward induction, see, for example, ROBERT GIBBONS, GAME THEORY FOR APPLIED ECONOMISTS (1992).

[6] The value of the scale factor A_m depends on the choice of units of measurement for the manufacturer's output and the relevant inputs purchased by the manufacturer. With no loss of generality,

assume that the manufacturer's cost of production can be reduced by purchasing inputs from multiple suppliers, so that the manufacturer's cost-minimization solution is to purchase inputs from several of the *N* suppliers.[7]

$S_m^j(W_m^1, W_m^2, ..., W_m^N)$ denotes supplier *j*'s share of the total volume of inputs purchased (from the *N* suppliers) by manufacturer *m*. Thus, the volume of inputs purchased by manufacturer *m* from supplier *j* is given by $S_m^j(W_m^1, W_m^2, ..., W_m^N) A_m Q_m$. We assume that the function S_m^j is decreasing in W_m^j and increasing in W_m^k for $k \neq j$. That is, supplier *j* will get a smaller share of the manufacturer's input purchases if it raises the price it charges the manufacturer for its input, and it will get a higher share if another supplier raises its price. We use $C_m(W_m^1, W_m^2, ..., W_m^N)$ to denote the marginal cost of production of manufacturer *m*. We assume that the function C_m is increasing in the input prices $(W_m^1, W_m^2, ..., W_m^N)$ faced by manufacturer *m*. (The marginal cost of production C_m also includes the incremental costs of other types of inputs.) We assume that the manufacturers' production technologies exhibit constant returns to scale, and thus the functions S_m^j and C_m do not depend on the level of output (Q_m).

2. *Competition Among Manufacturers in the Downstream Market*

Assuming a standard model of Bertrand competition with differentiated products, manufacturer *m* takes the equilibrium prices charged by the other manufacturers as given and chooses its price P_m to maximize its profit,

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we will set the value of A_m so that, at the pre-merger equilibrium, the manufacturer purchases from the upstream merging firm a quantity of input equal to the manufacturer's quantity of output. *See also infra* note 12.

[7] We assume each manufacturer faces a cost-minimization problem similar to that in Roman Inderst & Tommaso Valletti, *Incentives for Input Foreclosure*, 55 EUR. ECON. REV. 820 (2011).

$$(P_m - C_m) D_m (P_m, P_{-m}) \tag{A1}$$

where P_{-m} is the vector of equilibrium prices of all the other manufacturers and D_m is the demand function for the product sold by manufacturer *m*.[8] The first-order condition (FOC) for the price of manufacturer *m* to be profit-maximizing is:

$$D_m + (P_m - C_m)\frac{\partial D_m}{\partial P_m} = 0 \tag{A2}$$

Solving this equation for P_m, the equilibrium price strategy of manufacturer *m* is a function of the manufacturer's marginal cost, or $P_m(C_m)$.[9] Note that this pricing strategy is actually a function of the input prices that suppliers charge to manufacturer *m* since C_m depends on those prices (*see* Section I.A.1 above). For this reason, we sometimes write $P_m(W_m^1, W_m^2, ..., W_m^N)$ instead of $P_m(C_m)$.

[8] We focus on vertical mergers where the upstream and downstream merging firms are not vertically integrated. *See* SergeMoresi & Steven C. Salop, *vGUPPI: Scoring Unilateral Pricing Incentives in Vertical Mergers*, 79 ANTITRUST L.J. 185, 187 n.7, 188 n.11 (2013).

[9] This function implicitly also depends on the equilibrium prices of the other manufacturers. *See supra* note 4.

3. *Competition Among Suppliers in the Upstream Market*

Consider input supplier *n* and the price W_i^n that it charges to manufacturer *i*. In equilibrium, supplier *n* sets W_i^n to maximize its total profits holding all other prices constant, except for the price of manufacturer *i*, i.e., P_i.[10] We thus write the profit function of supplier *n* as:

$$(W_i^n - C_i^n) S_i^n (W_i^n) A_i D_i (P_i(W_i^n)) \;+\; \sum_{k \neq i}^{M} (W_k^n - C_k^n) S_k^n A_k D_k (P_i(W_i^n)) \qquad \text{(A3)}$$

where C_k^n is the marginal cost of supplier *n* of producing inputs for manufacturer *k*. This profit function adds up all the profits that supplier *n* obtains from all the manufacturers.

The FOC for W_i^n is:

$$(W_i^n - C_i^n) A_i \left(S_i^n \frac{\partial D_i}{\partial P_i} \frac{\partial P_i}{\partial W_i^n} + D_i \frac{\partial S_i^n}{\partial W_i^n} \right) + \; other\ terms \; = \; 0 \qquad \text{(A4)}$$

The first term in (A4) is negative and represents the loss from reduced sales to manufacturer *i* that supplier *n* would incur if it raised the price W_i^n that it charges to manufacturer *i*. Specifically, manufacturer *i* would reduce its purchases from supplier *n* for two reasons. First, manufacturer *i* would increase the price of its product by the amount $\partial P_i / \partial W_i^n$ and this would lead to a reduction in its volume of sales equal to $(\partial D_i / \partial P_i)(\partial P_i / \partial W_i^n)$. If manufacturer *i* were to hold supplier *n*'s share of its total input purchases constant (i.e., if S_i^n did not change), this would reduce supplier *n*'s input sales to manufacturer *i* by the amount $A_i S_i^n (\partial D_i / \partial P_i)(\partial P_i / \partial W_i^n)$. Second, manufacturer *i*

[10] The supplier takes the equilibrium prices charged by the other manufacturers as given because they do not observe the input price that the supplier is charging to the manufacturer under consideration.

actually would reduce S_i^n by the amount $\partial S_i^n / \partial W_i^n$ and this would lead to a further reduction in supplier *n*'s input sales to manufacturer *i* equal to $A_i D_i (\partial S_i^n / \partial W_i^n)$.

The "other terms" in (A4) are positive and represent two sources of profit gains to supplier *n* following an increase in the price charged to manufacturer *i*. First, supplier *n* would be earning a higher margin on the volume of inputs that manufacturer *i* would continue to purchase from supplier *n*. Second, supplier *n* would be selling larger volumes of inputs to the other manufacturers because some of the customers of manufacturer *i* would be switching to other manufacturers. To simplify the exposition, we do not show these profit gains explicitly in (A4) and instead denote them as "other terms." (These other terms do not matter for the derivation of the vGUPPIs, as will be shown below.)

In (perfect Bayesian) equilibrium, (A4) and (A2) are satisfied for all input prices and all output prices respectively.

B. Unilateral Effects of a Vertical Merger on Pricing Incentives

We analyze the potential unilateral effects on pricing incentives from a vertical merger of supplier 1 and manufacturer 1.

1. *vGUPPIu for the Upstream Merging Firm*

The vertical merger of supplier 1 and manufacturer 1 can give supplier 1 a unilateral incentive to raise the price of the input to manufacturer 2, a rival of the downstream merger partner. A similar unilateral incentive exists for each downstream rival that might be targeted with an input price increase.

We write the profit function of the merged firm as follows:

$$(P_1 - C_1) D_1(P_1, P_{-1}) \;+\; \sum_{k=1}^{M} (W_k^1 - C_k^1) S_k^1 A_k D_k(P_1, P_{-1}) \qquad \text{(A5)}$$

where $C_1 = C_1(W_1^1, \ldots, W_1^N)$ is the pre-merger marginal cost of manufacturer 1 since here we are holding constant both the prices faced by manufacturer 1 and the behavior of manufacturer 1. We analyze the incentives of supplier 1 to raise the input price to manufacturer 2. The post-merger FOC with respect to W_2^1 can be written as:

$$\begin{aligned} &(P_1 - C_1)\frac{\partial D_1}{\partial P_2}\frac{\partial P_2}{\partial W_2^1} + \\ &(W_2^1 - C_2^1)\left[A_2\left(S_2^1 \frac{\partial D_2}{\partial P_2}\frac{\partial P_2}{\partial W_2^1} + \frac{\partial S_2^1}{\partial W_2^1} D_2 \right)\right] + \ other\ terms\ =\ 0 \end{aligned} \tag{A6}$$

This FOC says that at the post-merger equilibrium the merged firm's incentive to increase W_2^1 must be zero. Instead of solving for the equilibrium of the model, we follow the GUPPI methodology and evaluate the merged firm's incentive to increase W_2^1, beginning at the pre-merger equilibrium. That is, we evaluate the left-hand side of (A6) at the pre-merger equilibrium strategies. Therefore, in (A6), the downstream price of each manufacturer *m* is the same function $P_m(W_m^1, W_m^2, \ldots, W_m^N)$ as pre-merger, and all input prices are the same prices as pre-merger.

Using (A4), the left-hand side of (A6) reduces to $(P_1 - C_1)(\partial D_1 / \partial P_2)(\partial P_2 / \partial W_2^1)$, since all the other terms add up to zero. This term is positive and thus the merged firm has an incentive to raise W_2^1 above its pre-merger level. Intuitively, an increase in the price charged to manufacturer 2 (W_2^1) leads manufacturer 2 to raise the price of its output (P_2) and that in turn leads to an increase in the sales of manufacturer 1 (D_1). This has a positive effect on the profits of manufacturer 1 (the downstream merging firm) and thus creates upward pricing pressure on supplier 1 (the upstream merger partner) to raise the input price W_2^1 charged to manufacturer 2.

The GUPPI methodology quantifies this upward pricing pressure as follows. One first divides the incentive effect identified in the previous paragraph by the expression in square brackets in (A6). This allows us to interpret and measure the upward pricing pressure on the input price W_2^1 as being equivalent to an increase in supplier 1's marginal cost of supplying the input to manufacturer 2. It then expresses this "equivalent input marginal cost increase" as a percentage of the input price.

It follows that *vGUPPIu* (i.e., the vGUPPI for the input price that supplier 1 charges to manufacturer 2) is given by:

$$vGUPPIu = \frac{(P_1 - C_1)\dfrac{\partial D_1}{\partial P_2}\dfrac{\partial P_2}{\partial W_2^1}}{\left(-S_2^1 \dfrac{\partial D_2}{\partial P_2}\dfrac{\partial P_2}{\partial W_2^1} - \dfrac{\partial S_2^1}{\partial W_2^1} D_2\right) A_2 W_2^1} \qquad \text{(A7)}$$

This is the standard GUPPI ratio described in the 2010 Merger Guidelines. The numerator is the "value of sales diverted" to the merger partner, that is, the profit gained by manufacturer 1 following an increase in the input price charged to manufacturer 2. The denominator is the "lost revenues attributable to the reduction in unit sales [to manufacturer 2] resulting from the input price increase [to manufacturer 2]."[11]

Dividing both the numerator and the denominator by $-(\partial D_2 / \partial P_2)(\partial P_2 / \partial W_2^1)$ and then using the FOC for P_2 (i.e., $P_2 - C_2 = -D_2 /(\partial D_2 / \partial P_2)$), one obtains:

$$vGUPPIu = \frac{DR_{21} M_1 P_1}{(1 + M_2 E_{SR} / E_P) S_2^1 A_2 W_2^1} \qquad \text{(A8)}$$

where $DR_{21} = -(\partial D_1 / \partial P_2)/(\partial D_2 / \partial P_2)$ is the diversion ratio from manufacturer 2 to manufacturer 1, $M_m = (P_m - C_m)/P_m$ is the percentage profit margin of manufacturer *m*,

[11] U.S. Dep't of Justice & Fed. Trade Comm'n, Horizontal Merger Guidelines § 6.1 (2010), *available at* http://www.justice.gov/atr/public/guidelines/hmg-2010.pdf.

$E_{SR} = -(\partial S_2^1 / \partial W_2^1)(W_2^1 / S_2^1)$ is the elasticity of supplier 1's share of manufacturer 2's total input purchases with respect to an increase in the input price W_2^1 charged by supplier 1 to manufacturer 2, and $E_P = (\partial P_2 / \partial W_2^1)(W_2^1 / P_2)$ is the elasticity of the price of the output of manufacturer 2 with respect to an increase in the input price W_2^1 charged by supplier 1 to manufacturer 2.[12]

Note that the *vGUPPIu* formula in (A8) depends on the pre-merger profit margin ($P_1 - C_1$) of the downstream merger partner. This could suggest that a higher profit margin should be used in the *vGUPPIu* formula if manufacturer 1 will obtain the input of supplier 1 at cost after the merger. However, this would not be correct because the cost savings of manufacturer 1 will be offset by a corresponding reduction in the revenues of supplier 1.[13]

2. vGUPPId for the Downstream Merging Firm

The vertical merger also affects the unilateral pricing incentives of merging manufacturer 1. The merger creates two opposing effects. On the one hand, manufacturer 1 will take into account that an increase in the price of its output, P_1, would

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[12] Equations (1) and (5) in the article text are derived from (A8). With no loss of generality, we assume that units of measurement are such that, at the pre-merger equilibrium, $S_2^1 A_2 = 1$, i.e., each targeted rival purchases a quantity of input from the upstream merging firm equal to the quantity of output that it produces. This simply means that the input price W_2^1 must be calculated as the cost of the input purchased from the upstream merging firm per unit of output produced by the targeted rival.

[13] As explained above in Parts I.C and II.C of the article, the merger may lead to a reduction in the marginal cost of production of manufacturer 1 for two reasons. First, manufacturer 1 will obtain the input of supplier 1 at a price equal to marginal cost (instead of a marked-up price). This is the standard EDM effect of vertical mergers. The *vGUPPIu* accounts for this effect. Second, there can be an additional reduction in the marginal cost of production of the merged firm if manufacturer 1 can substitute inputs purchased from other suppliers with inputs produced by the upstream merger partner. Accounting for this second type of "vertical efficiency" would increase the *vGUPPIu*. This would introduce a "perverse effect" of this type of efficiency that is similar to that discussed in Farrell & Shapiro. Joseph Farrell & Carl Shapiro, *Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition*, B.E. J. THEOR. ECON., vol. 10, no. 1, art. 9, at 23 (2010), http://www.bepress.com/bejte/vol10/iss1/art41.

lead other manufacturers to increase output and thus purchase more inputs from supplier 1 (the upstream merger partner). This creates upward pricing pressure on P_1. On the other hand, to the extent that manufacturer 1 purchases inputs from supplier 1 at a marginal price that exceeds marginal cost, manufacturer 1 will take into account that a reduction in the price of its output would lead it to increase output and purchase more inputs from supplier 1, thereby increasing the profits of supplier 1. This creates downward pricing pressure on P_1.

The profit function of the merged firm can be written as:

$$(P_1 - C_1)D_1(P_1, P_{-1}) \ + \ \sum_{k=2}^{M}(W_k^1 - C_k^1)S_k^1 A_k D_k(P_1, P_{-1}) \qquad \text{(A9)}$$

where $C_1 = C_1(C_1^1, W_1^2, ..., W_1^N)$ is the post-merger marginal cost of manufacturer 1 since supplier 1 will provide the input to manufacturer 1 at cost, i.e., $W_1^1 = C_1^1$. For later use, we denote by ΔC_1 the post-merger reduction in the marginal cost of production of manufacturer 1:

$$\Delta C_1 = C_1(W_1^1, W_1^2, ..., W_1^N) - C_1(C_1^1, W_1^2, ..., W_1^N) \qquad \text{(A10)}$$

The first-order condition with respect to P_1 reads:

$$D_1 \ + \ (P_1 - C_1)\frac{\partial D_1}{\partial P_1} \ + \ \sum_{k=2}^{M}(W_k^1 - C_k^1)S_k^1 A_k \frac{\partial D_k}{\partial P_1} \ = \ 0 \qquad \text{(A11)}$$

or equivalently:

$$D_1 \ + \ (P_1 - C_1 - \sum_{k=2}^{M} S_k^1 A_k DR_{1k} M_k^1 W_k^1)\frac{\partial D_1}{\partial P_1} \ = \ 0 \qquad \text{(A12)}$$

where $DR_{1k} = -(\partial D_k / \partial P_1)/(\partial D_1 / \partial P_1)$ is the diversion ratio from manufacturer 1 to manufacturer *k* (following a unilateral price increase by manufacturer 1) and

$M_k^1 = (W_k^1 - C_k^1)/W_k^1$ is the percentage profit margin that supplier 1 earns on sales to manufacturer *k*.

Comparing (A2) and (A12) (evaluated at the pre-merger equilibrium point), it follows that the merger affects the pricing incentives of manufacturer 1 (i.e., the merger partner that operates downstream) in two ways. First, the merger increases the "effective marginal cost" of manufacturer 1 by the amount $\sum_{k=2}^{M} S_k^1 A_k DR_{1k} M_k^1 W_k^1$. Intuitively, suppose that manufacturer 1 unilaterally reduces its price and increases output by 1 unit. This increases the production cost of manufacturer 1 (by C_1) and reduces the profits of supplier 1 (i.e., the merger partner that operates upstream). This is because the increase in output of manufacturer 1 leads to a reduction in output by manufacturer *k* equal to DR_{1k} units, and therefore to a reduction $S_k^1 A_k DR_{1k}$ in the amount of input that manufacturer *k* purchases from supplier 1. This in turn decreases the profit of supplier 1 by an amount equal to $S_k^1 A_k DR_{1k} M_k^1 W_k^1$. Since a similar effect occurs with respect to each of the rivals of manufacturer 1, the total loss of supplier 1 is equal to $\sum_{k=2}^{M} S_k^1 A_k DR_{1k} M_k^1 W_k^1$. This loss to the upstream merger partner becomes an "opportunity cost" of expanding manufacturer 1's sales post-merger, and thus creates upward pricing pressure on the output price P_1 of manufacturer 1.

Second, the merger might reduce the marginal cost of production faced by manufacturer 1 by the amount ΔC_1, what is commonly referred to as EDM (i.e., elimination of double marginalization). Merger-specific EDM creates a downward pricing pressure on the output price P_1 of manufacturer 1.

It then follows that the vertical GUPPI for the price of manufacturer 1, which we denote *vGUPPId*, is equal to:

$$vGUPPId \;=\; vGUPPId1 \;-\; \frac{\Delta C_1}{P_1} \tag{A13}$$

where

$$vGUPPId1 \;=\; \sum_{k=2}^{M} S_k^1 A_k DR_{1k} M_k^1 W_k^1 / P_1 \tag{A14}$$

is the vertical GUPPI for the price of manufacturer 1 in the absence of EDM.

This analysis can be simplified by (i) assuming that supplier 1 happens to charge the same price to all the rivals of manufacturer 1, i.e., $W_k^1 = W_{-1}^1$, (ii) assuming that supplier 1 earns the same margin on all the sales to the rivals of manufacturer 1, i.e., $M_k^1 = M_{-1}^1$, and (iii) defining the diversion ratio from manufacturer 1 to supplier 1 equal to $DR_1^1 = \sum_{k=2}^{M} S_k^1 A_k DR_{1k}$. In this case, (A13) reduces to:

$$vGUPPId \;=\; \frac{DR_1^1 M_{-1}^1 W_{-1}^1 \;-\; \Delta C_1}{P_1}\;. \tag{A15}$$

In the main text of the article, we use (A15) and measure each of M_{-1}^1 and W_{-1}^1 by its average value across all the rivals of manufacturer 1. In addition, we approximate the reduction in manufacturer 1's marginal cost using:

$$\Delta C_1 \;=\; (S_1^1 + \Delta S_1^1) A_1 M_1^1 W_1^1 \tag{A16}$$

where ΔS_1^1 is the increase in supplier 1's share of the total inputs used by manufacturer 1 post-merger. This implicitly assumes that pre-merger all the suppliers charge the same input price to manufacturer 1, i.e., $W_1^{-1} = W_1^1$. Thus, manufacturer 1 saves an amount equal to $W_1^1 - C_1^1$ on each unit of input obtained from supplier 1 post-merger.[14]

The *vGUPPId1* formula in (3) corresponds to (A15) with $\Delta C_1 = 0$ (i.e., there is no EDM and no input substitution), while the *vGUPPId2* formula in (4) corresponds to (A15) and (A16) under the assumption that $\Delta S_1^1 = 0$ (i.e., there is EDM but there is no input substitution).

II. DERIVATION OF THE vGUPPIS WHEN THERE IS INPUT SUBSTITUTION

When there is input substitution, several of the derivations change.[15]

A. EQUATION (5)

Input substitution breaks the equality between DR_{UD} and DR_{RD} as follows:

$$DR_{UD} = \frac{DR_{RD} \times E_P / M_R}{E_P / M_R + E_{SR}}. \tag{B1}$$

Intuitively, from the Lerner condition, the demand elasticity faced by the downstream rival is equal to $1/M_R$. It follows that E_P / M_R measures the elasticity of the rival's total purchases of inputs with respect to the input price paid to the upstream merging firm. The numerator of (B1) thus represents the volume of sales gained by the downstream merging firm; it depends on both the extent to which the targeted rival will lose sales (E_P / M_R) and the extent to which lost sales by the targeted rival are diverted to the downstream merging firm (DR_{RD}). The denominator represents the volume of sales (to

[14] Equation (A15) is an approximation because it assumes that the downstream merging firm can substitute some of the inputs purchased from other suppliers with inputs from the upstream merger partner without incurring any additional costs.

[15] In this Appendix, we use the notation from the article text, that is, the subscript "*U*" refers to the upstream merging firm and the subscripts "*D*" and "*R*" refer to the downstream merging firm and targeted rival, respectively.

the targeted rival) lost by the upstream merging firm; the first term (E_P / M_R) measures the extent to which the targeted rival will reduce its purchases from the upstream merging firm because the targeted rival will raise price and thus reduce output; the second term (E_{SR}) measures the extent to which the targeted rival will reduce its purchases from the upstream merging firm because the targeted rival will substitute inputs from the upstream merging firm with inputs from other suppliers. Equations (1) and (B1) then imply (5).

B. EQUATIONS (6) AND (7)

All else equal, the percentage increase in the input price charged to the targeted rival would be equal to $vGUPPIu \times PTR_U$.[16] If there is input substitution, however, the expression $vGUPPIu \times PTR_U \times E_{SR}$ is the percentage reduction in the upstream merging firm's share (S_{UR}) of the targeted rival's input purchases. Therefore, post-merger the upstream firm's share of the targeted rival's total input purchases falls to the level S_{UR}^{post} given by:

$$S_{UR}^{post} = (1 - vGUPPIu \times PTR_U \times E_{SR}) \times S_{UR} \qquad \text{(B2)}$$

and the rival's marginal cost rises by $\Delta C_R = vGUPPIu \times PTR_U \times W_R \times S_{UR}^{post} / S_{UR}$.[17] Equation (B2) can be written as equation (7). The *vGUPPIr* is by definition equal to $\Delta C_R / P_R$, which leads to equation (6).

C. EQUATIONS (4) AND (8)

As explained above, the *vGUPPId* formula (A13) can be written as:

[16] *See* Sonia Jaffe & E. Glen Weyl, *The First-Order Approach to Merger Analysis*, 5 AM. ECON. J.: MICROECON. 188 (2013).

[17] This is an approximation because it assumes that the targeted downstream rival can substitute some of the inputs purchased from the upstream merging firm with inputs from other suppliers without incurring any additional costs. In addition, (B2) is a linear approximation.

$$vGUPPId = vGUPPId1 - \frac{\Delta C_D}{P_D} \qquad \text{(B3)}$$

where *vGUPPId1* is given in equation (3) and ΔC_D is given in (A16), which we rewrite with different notation as follows:[18]

$$\Delta C_D = (1 + \frac{\Delta S_{UD}}{S_{UD}}) \times M_{UD} \times W_D . \qquad \text{(B4)}$$

If there is no input substitution, $\Delta S_{UD} = 0$. Thus, (B3) and (B4) lead to (4). If there is input substitution, we use the approximation $\Delta S_{UD} = M_{UD} \times E_{SD} \times S_{UD}$. Combining this with (B3) and (B4) leads to the equation for *vGUPPId3* in (8).

D. Equation (9)

Assuming symmetric downstream firms, the profit function (A3) of the upstream merging firm can be written as:

$$(W - C)S(W)AD(P(W)). \qquad \text{(B5)}$$

Here, W denotes the input price charged by the upstream firm to each downstream firm, C denotes the upstream firm's marginal cost, $S(W)$ is the upstream firm's share of the total input purchases of the downstream firms, $P(W)$ is the output price charged by each downstream firm, $D(P)$ is total output of the downstream firms, and A is a scale factor.

The first-order condition of the maximization of (B5) with respect to W can be written as equation (9).

[18] *Supra* note 12.